Imitation Learning
&
Behavioral Cloning

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CS/Stat 184: Introduction to Reinforcement Learning
Fall 2022

Today

- HW4 will be posted today. (Please start early!)
- Recap++
  - an example + Proximal Policy Optimization (PPO)
- Today: Theory
  1. ~~Overview~~ of PG, problems+successes
  2. Behavior Cloning

Recap++

Some Helpful Notation: Visitation Measures

$s_0 \sim \mu$.

- Visitation probability at time h: $\mathbb{P}_h(s_h, a_h \mid \mu, \pi)$
  (recall that we absorb h, into the state, i.e. $s \leftarrow (s, h)$)
- Average Visitation Measure:

$$d^{\pi}_{\mu}(s, a) = \frac{1}{H} \sum_{h=0}^{H-1} \mathbb{P}_h(s, a \mid \mu, \pi)$$

- With this def, we have:

$$J(\theta) := E_{s_0 \sim \mu_0}\left[V^{\pi_\theta}(s_0)\right]$$

$$= E\left[\sum_{h=0}^{H-1} r(s_h, a_h) \,\middle|\, \mu_0, \pi_\theta\right] = H \, E_{s,a \sim d^{\pi}_{\mu}}\left[r(s,a)\right]$$

$$= H \cdot \underline{\mathbb{E}_{s \sim d^{\pi_\theta}_{\mu}}} E_{a \sim \pi_\theta(s)}\left[r(s, a)\right]$$

TRPO

At iteration t, with π_{θ_t} at hand, we compute θ_{t+1} as follows:

$$\max_{\theta} H \cdot \mathbb{E}_{s \sim d_\mu^{\pi_{\theta_t}}} \left[\mathbb{E}_{a \sim \pi_\theta(s)} A^{\pi_{\theta_t}}(s, a) \right]$$

$$\text{s.t., } KL\left(\rho_{\pi_{\theta_t}} | \rho_{\pi_\theta} \right) \leq \delta$$

We want to maximize local advantage against π_{θ_t}, but we want the new policy to be close to π_{θ_t} (in the KL sense)

How we can actually do the optimization here?
After all, we don't even know the analytical form of trajectory likelihood…

NPG derived from TRPO:

We did second-order Taylor expansion on the KL constraint, and we get:

$$\frac{1}{H} KL\left(\rho_{\pi_{\theta_t}} | \rho_{\pi_\theta}\right) \approx \frac{1}{2}(\theta - \theta_t)^\top F_{\theta_t}(\theta - \theta_t)$$

$$F_{\theta_t} := \mathbb{E}_{s,a \sim d_\mu^{\pi_{\theta_t}}}\left[\nabla_\theta \ln \pi_{\theta_t}(a | s)\left(\nabla_\theta \ln \pi_{\theta_t}(a | s)\right)^\top\right] \in \mathbb{R}^{dim_\theta \times dim_\theta}$$

This leads to the following simplified constrained optimization:

$$\max_\theta \nabla_\theta J(\pi_{\theta_t})^\top \left(\theta - \theta_t\right)$$

s.t. $(\theta - \theta_t)^\top F_{\theta_t}(\theta - \theta_t) \leq \delta$

Algorithm: Natural Policy Gradient

Initialize θ_0

For t = 0, …

Estimate PG $\widehat{\nabla_\theta J(\pi_{\theta_t})}$

Estimate Fisher info-matrix $F_{\theta_t} := \widehat{\mathbb{E}}_{s,a\sim d_\mu^{\pi_{\theta_t}}} \nabla_\theta \ln \pi_{\theta_t}(a \mid s)\left(\nabla_\theta \ln \pi_{\theta_t}(a \mid s)\right)^\top$

Natural Gradient Ascent: $\theta_{t+1} = \theta_t + \eta \widehat{F}_{\theta_t}^{-1} \widehat{\nabla}_\theta J(\pi_{\theta_t})$

Using a tuned η or using $\eta = \sqrt{\dfrac{\delta}{\nabla_\theta J(\pi_{\theta_t})^\top F_{\theta_t}^{-1} \nabla_\theta J(\pi_{\theta_t})}}$

(We will implement it in HW4 on Cartpole)

Example of Natural Gradient on 1-d problem: 2 actions, 1 state

$$(\pi_\theta[1], \pi_\theta[2]) := \left(\frac{\exp(\theta)}{1 + \exp(\theta)}, \frac{1}{1 + \exp(\theta)} \right)$$

$$J(\theta) = 100 \cdot \pi_\theta[1] + 1 \cdot \pi_\theta[2]$$

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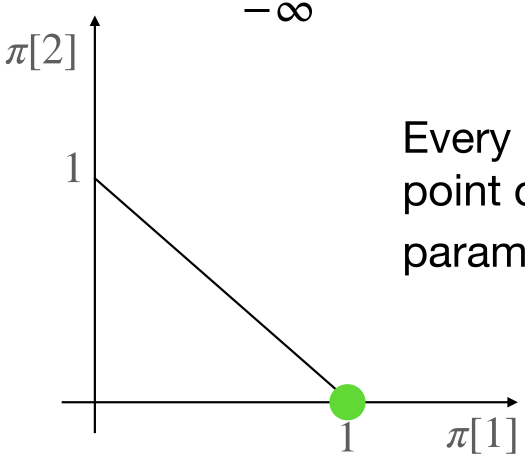

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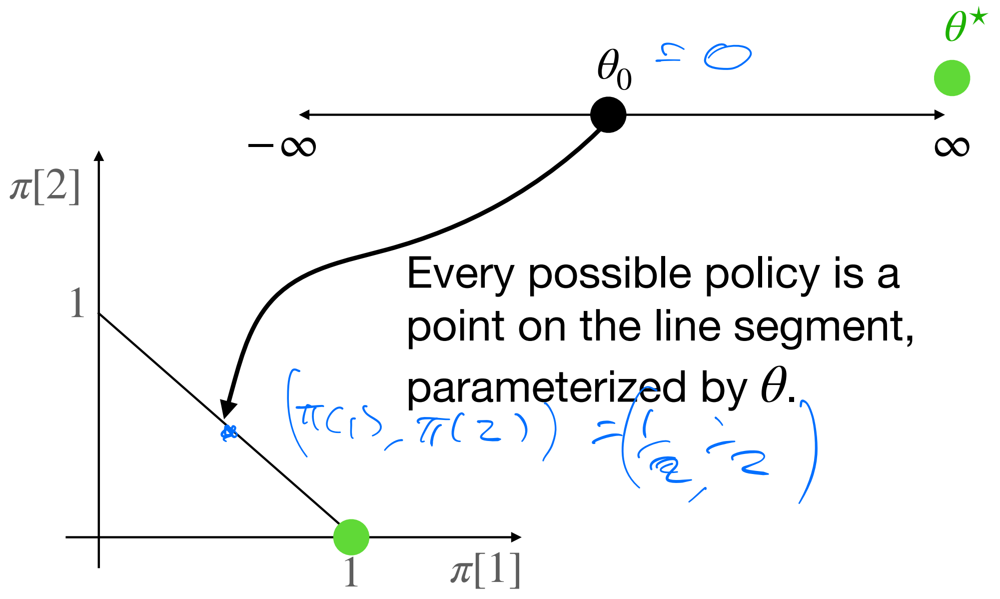

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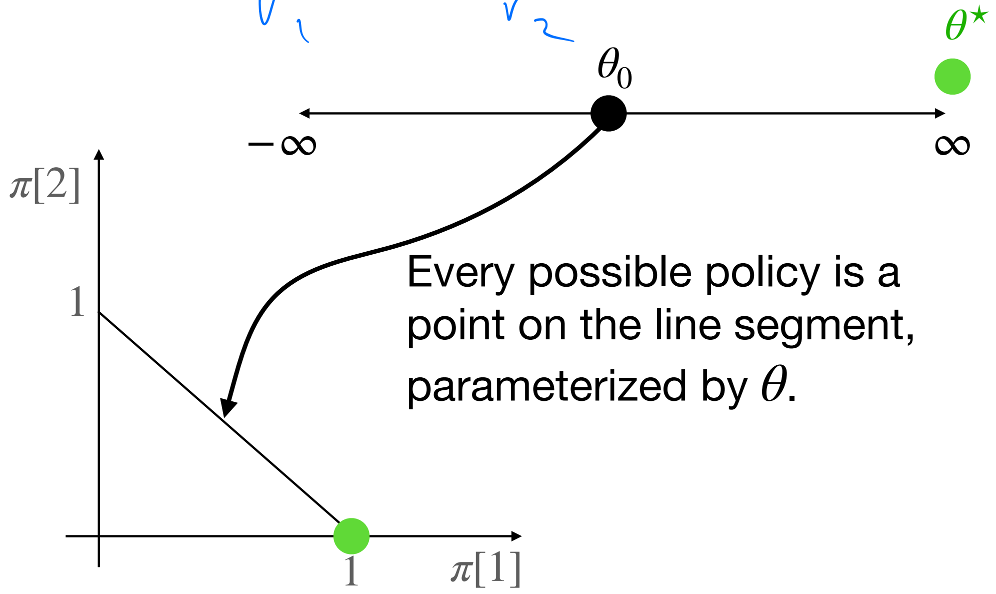
$\pi_\theta'(1) = \pi_\theta(1)(1-\pi_\theta(1))$

$$(\pi_\theta[1], \pi_\theta[2]) := \left(\frac{\exp(\theta)}{1+\exp(\theta)}, \frac{1}{1+\exp(\theta)}\right)$$

$(r_1 - r_2) \cdot \pi_\theta(1)(1-\pi_\theta(1))$

Gradient: $J'(\theta) = \dfrac{99\exp(\theta)}{(1+\exp(\theta))^2}$

$$J(\theta) = \underset{r_1}{100} \cdot \pi_\theta[1] + \underset{r_2}{1} \cdot \pi_\theta[2]$$



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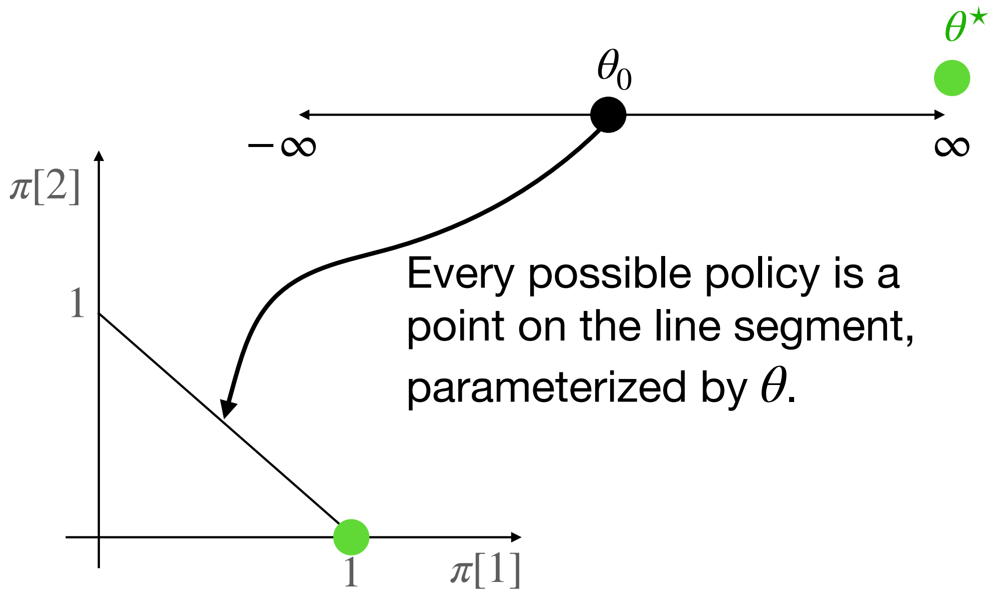

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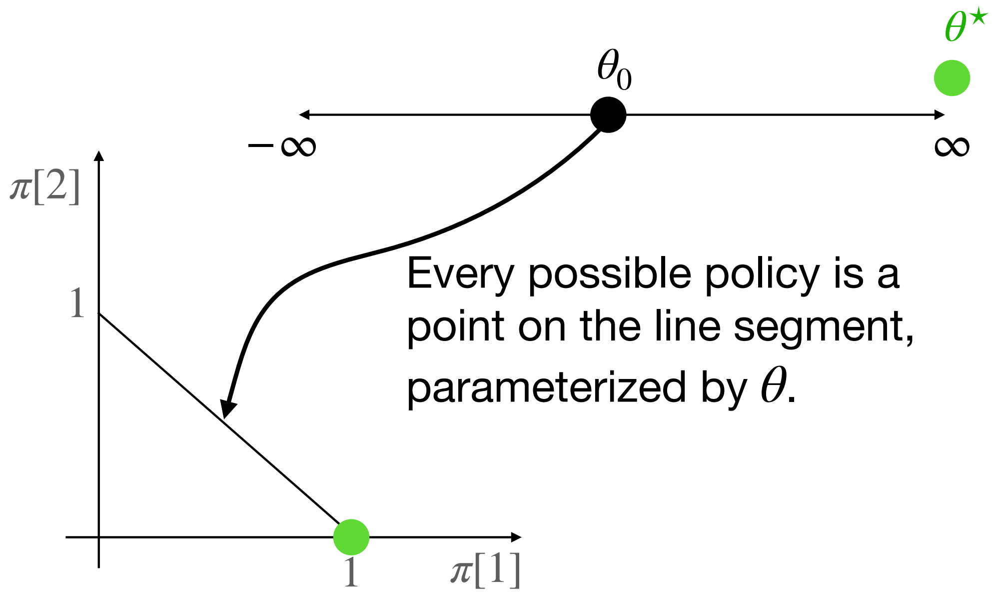

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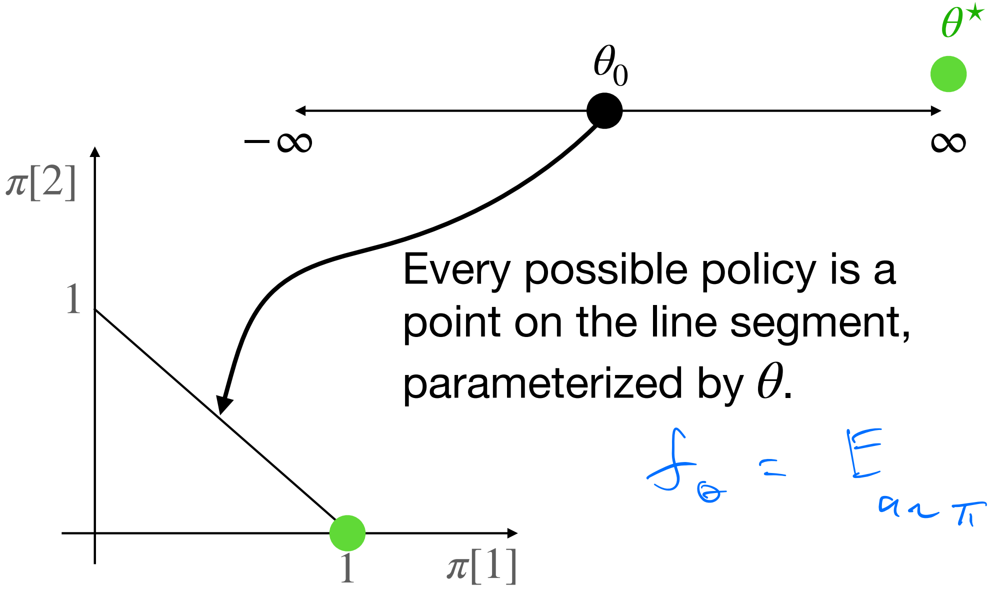

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$$f_\theta = E_{a\sim\pi}\left[\left(\left(\log \pi_\theta(a)\right)'\right)^2\right]$$

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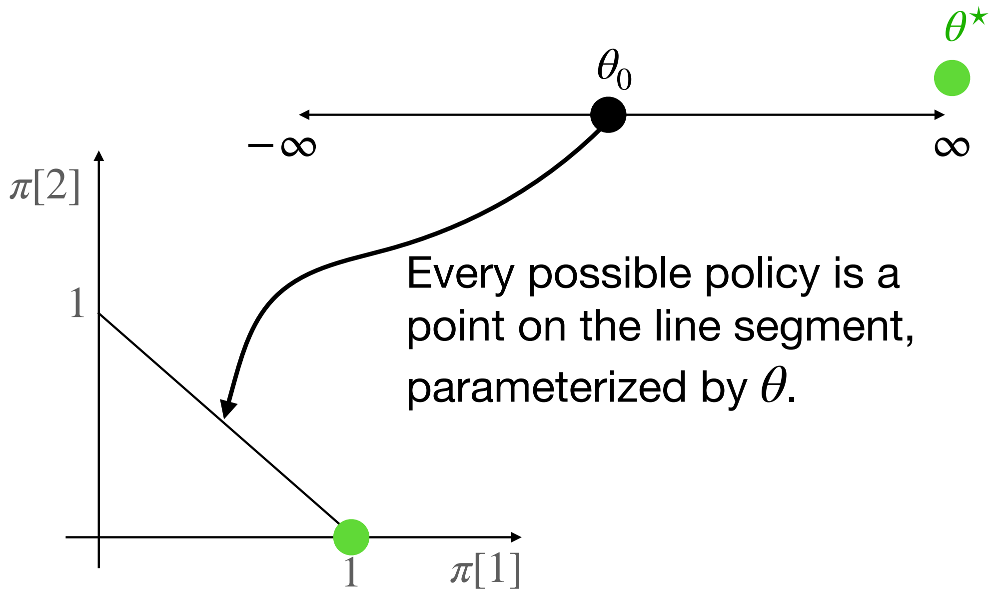

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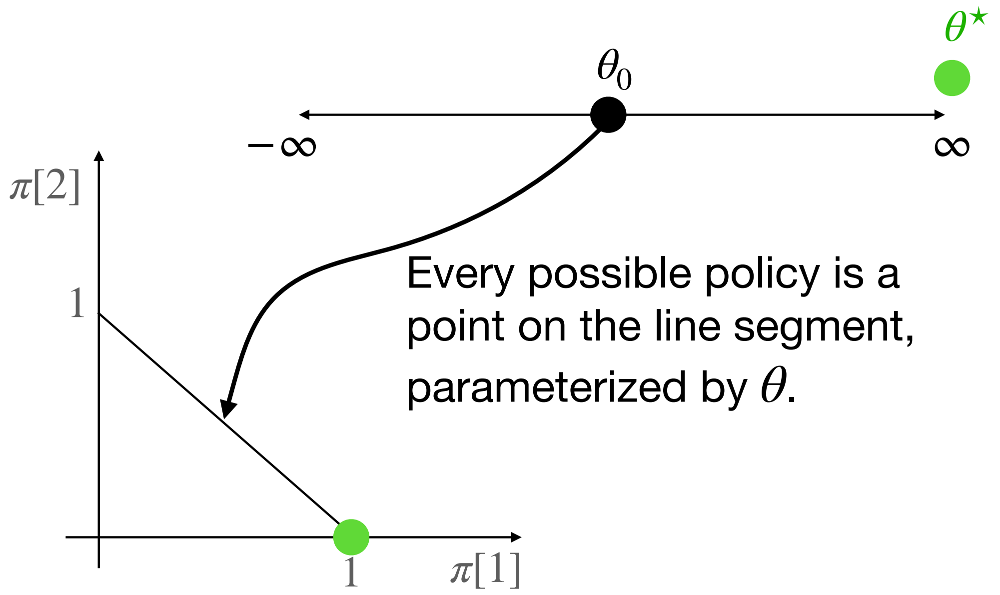

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$(r_1 - r_2)$

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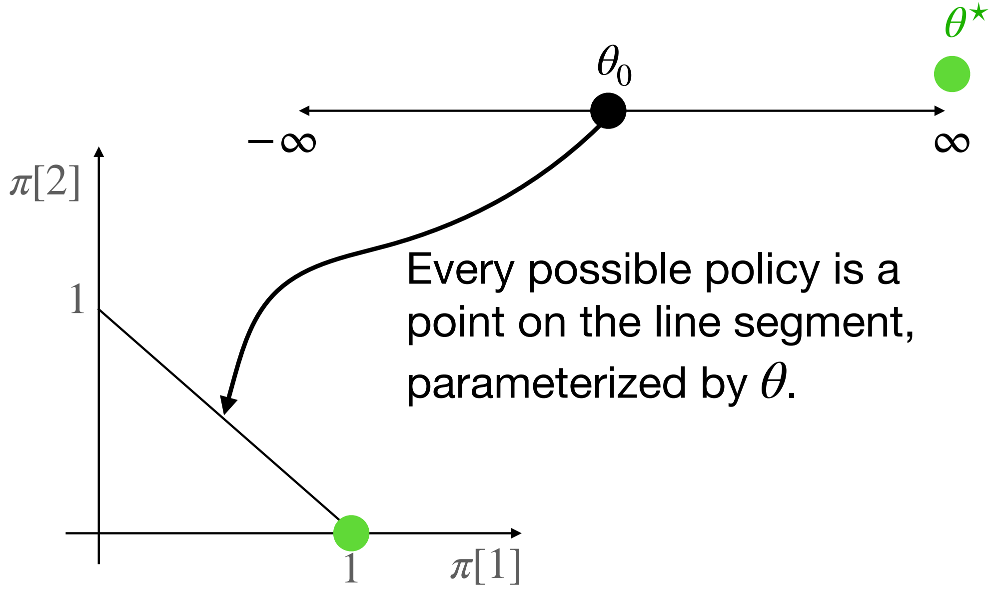

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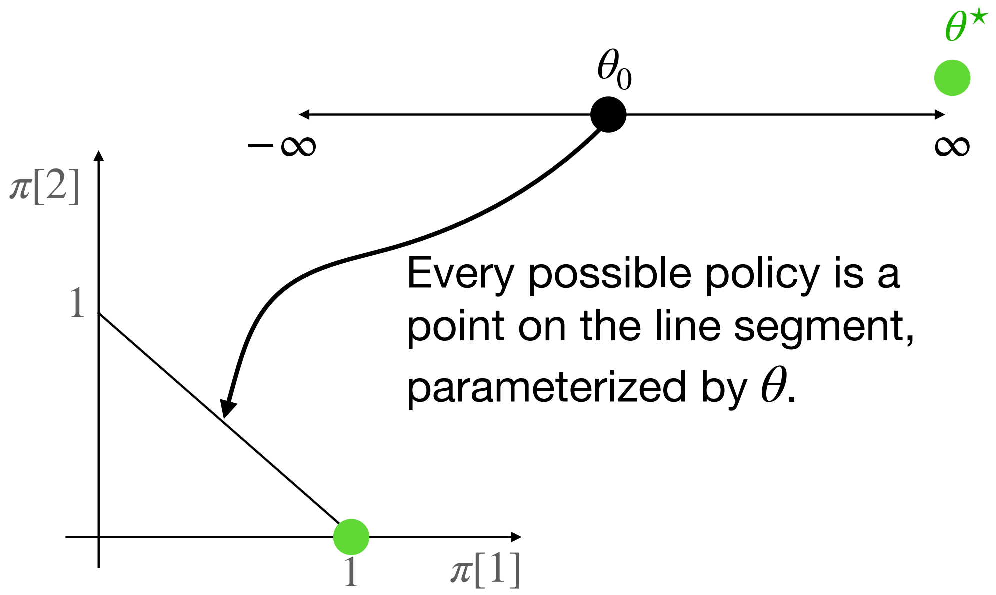

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Proximal Policy Optimization (PPO):
A computationally fast extension of NPG:

Given an current policy π^t, we perform policy update to π^{t+1}

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PPO: Perform a few steps of mini-batch SGA on $\ell(\theta)$ to approximate $\arg\max_\theta \ell(\theta)$

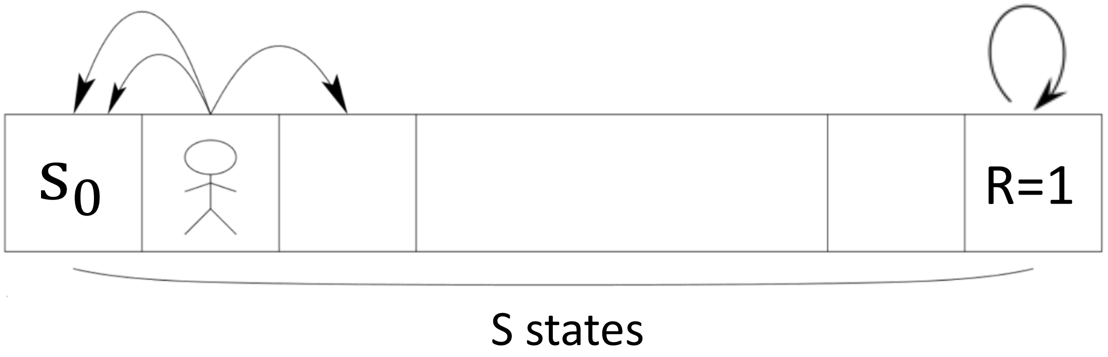
Today:

Optimality in Markov Decision Processes

Outline:

1. Exploration and the starting measure μ
2. Theory: (natural) policy gradients vs fitted Dynamic programming
3. Behavioral Cloning

“Lack of Exploration” leads to Optimization and Statistical Challenges



Thrun ‘92

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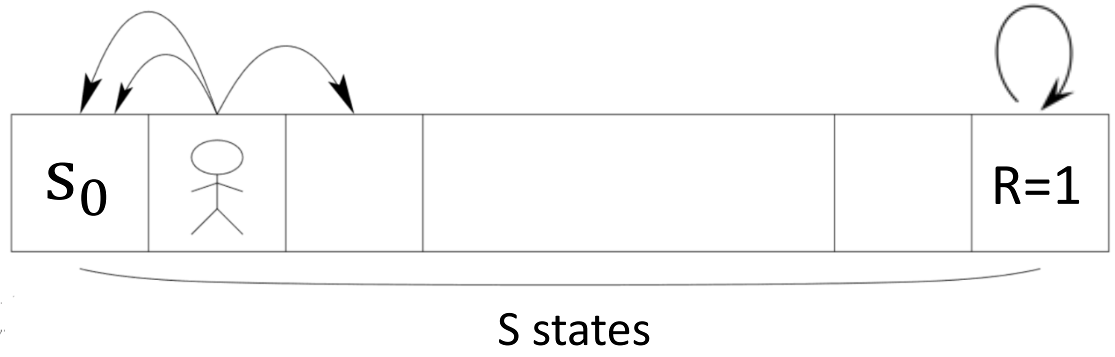

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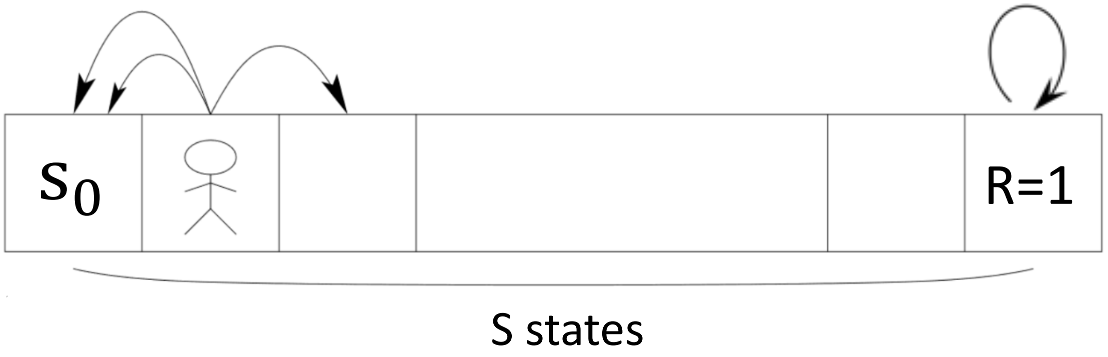

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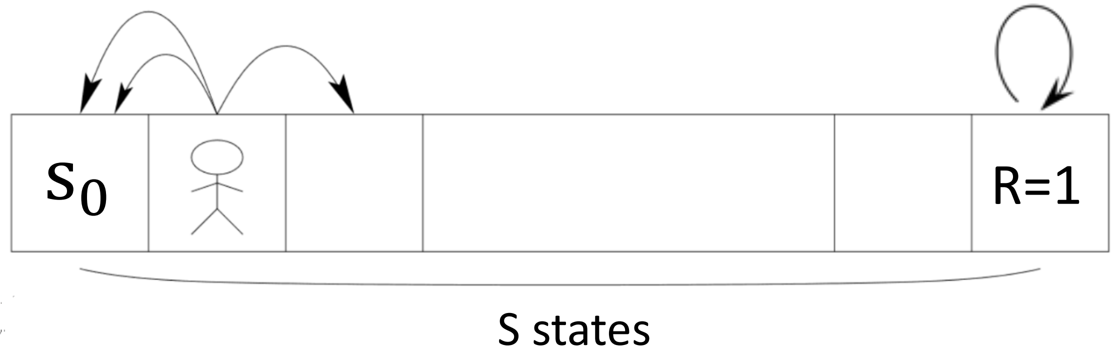

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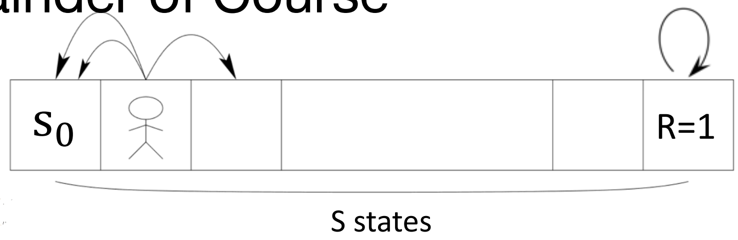
- Implications:
  - Any sample based policy iteration approach (starting with this policy) requires $O(3^{|S|})$ trajectories to make progress at the very first step.
  - Same for any sample based PG method.
  - Related: even if we had exact gradients, the “landscape” is such that these gradients are exponentially small, at randomly initialized policy (see AJKS Ch 11).

Implications/Comments/Remainder of Course



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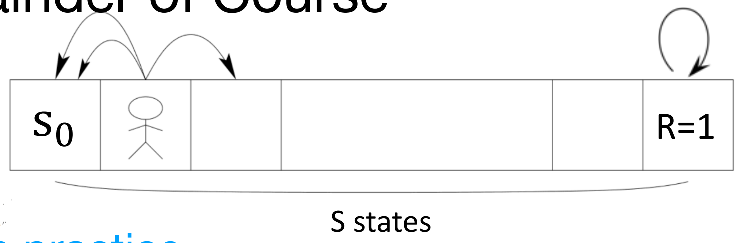

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- We can try to make the distribution μ to have better coverage.
  - For small problems, μ being uniform would make all these issues go away. (for large problems, μ being uniform may not help at all. Why?)
  - Ideally, μ having support on where a good policy tends to visit is helpful (sometimes we can't design μ)

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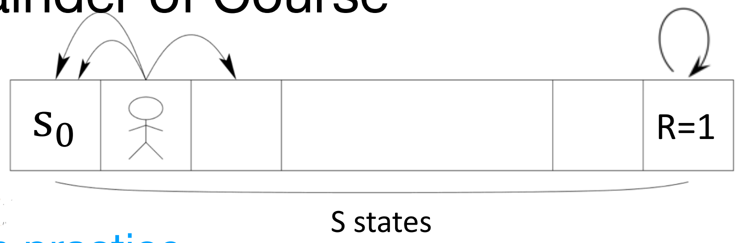

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- Course:
  - A little theory with regards to μ and PG. (today) PG has better guarantees than approx DP methods (in terms of μ).
  - Imitation learning (starting today). An expert gives us samples from a "good" μ.
  - Explicit Exploration: for the "tabular case" (we will mix UCB with VI!)

Outline:

1. Exploration and the starting measure μ
2. Theory: (natural) policy gradients vs fitted Dynamic programming
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Let's compare fitted DP and PG for "Linear" Parameterizations of Q-functions and Policies

Feature vector $\phi(s, a) \in \mathbb{R}^d$, and
parameter $\theta \in \mathbb{R}^d$

1. Linear Functions

$$f_\theta(s, a) = \theta^\top \phi(s, a)$$

1. Softmax linear Policy

$$\pi_\theta(a \mid s) = \frac{\exp(\theta^\top \phi(s, a))}{\sum_{a'} \exp(\theta^\top \phi(s, a'))}$$

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- Let $s_0, a_0 \sim \mu$ now be the starting "state-action" distribution. $J(\pi) = E_{s_0,a_0 \sim \mu}[Q^\pi(s, a)]$
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- Approximation error: For all policies, suppose that for all π,

$\min_\theta E_{s,a\sim\mu}\left[\left(Q^\pi(s,a) - \theta^\top \phi(s,a)\right)^2\right] \le \delta$, and $\min_\theta \|Q^\pi - \theta^\top \phi\|_\infty \le \delta_\infty$

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  - δ: the average case supervised learning error (reasonable to expect this can be made small)
    δ_∞: the worse case error (often unreasonable to expect to be small)

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  - δ: the average case supervised learning error (reasonable to expect this can be made small)
    δ_∞: the worse case error (often unreasonable to expect to be small)

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- NPG has the same guarantee.
- NPG also has a stronger guarantee: Suppose μ has "reasonable support" on where $\pi^\star$ tends to visit, i.e. suppose:

$$\max_{s,a}\left(\frac{d_\mu^{\pi^\star}(s,a)}{\mu(s,a)}\right) \leq C$$

  then NPG will return a policy with sub-optimality determined by C and the average case error δ:

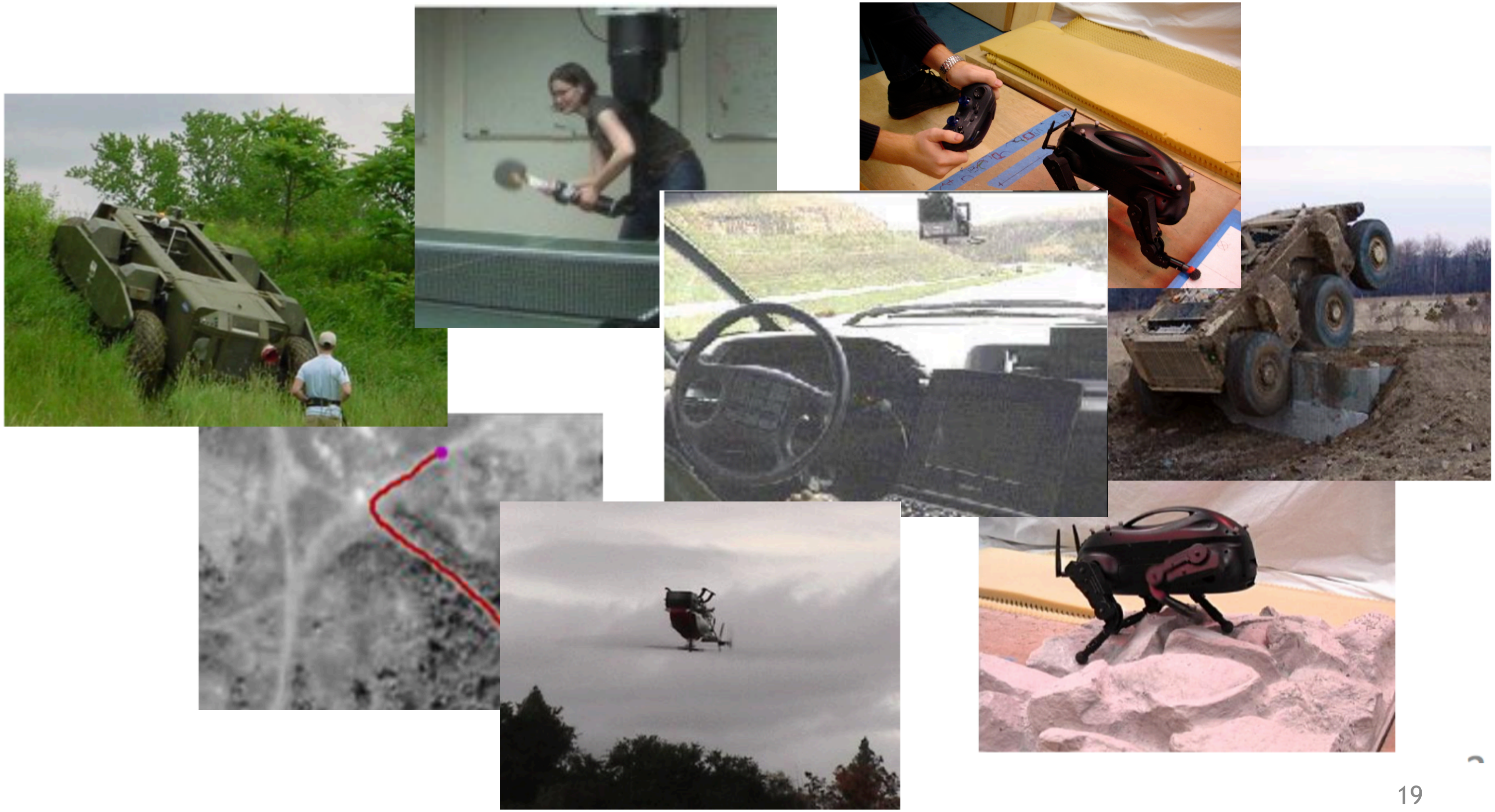
$$J(\pi^{NPG}) \geq J(\pi^\star) - \epsilon_{stat} - 2H^2 C\delta$$

Outline:

1. Exploration and the starting measure μ
2. Theory: (natural) policy gradients vs fitted Dynamic programming
3. Behavioral Cloning

3a. Introduction of Imitation Learning

Imitation Learning



Imitation Learning

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Imitation Learning

Expert Demonstrations

Imitation Learning

Expert Demonstrations

Machine Learning Algorithm

- SVM
- Gaussian Process
- Kernel Estimator
- Deep Networks
- Random Forests
- LWR
- …

Imitation Learning

Expert Demonstrations → Machine Learning Algorithm → Policy π

- SVM
- Gaussian Process
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- …

Maps *states* to actions

Learning to Drive by Imitation

[Pomerleau89, Saxena05, Ross11a]

Input:

Camera Image

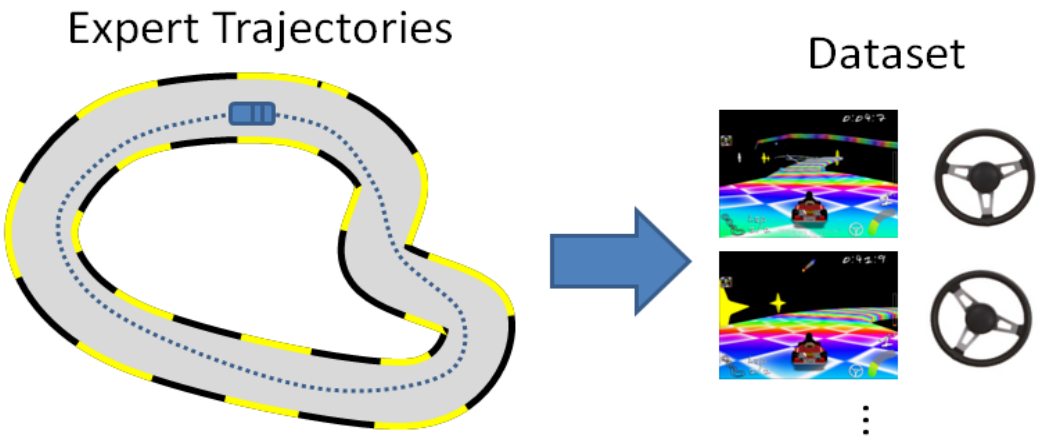
Policy

Output:

Steering Angle
in [-1, 1]

[Widrow64,Pomerleau89]

Supervised Learning Approach: Behavior Cloning



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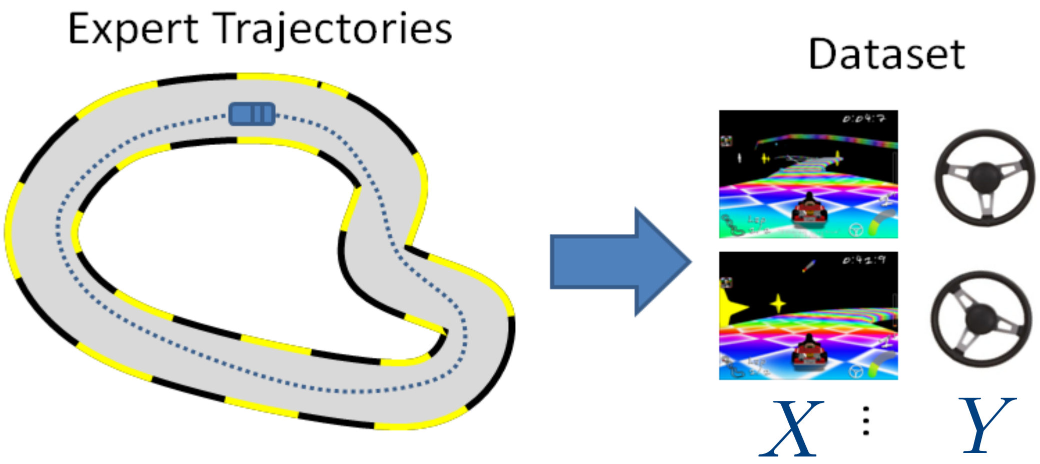

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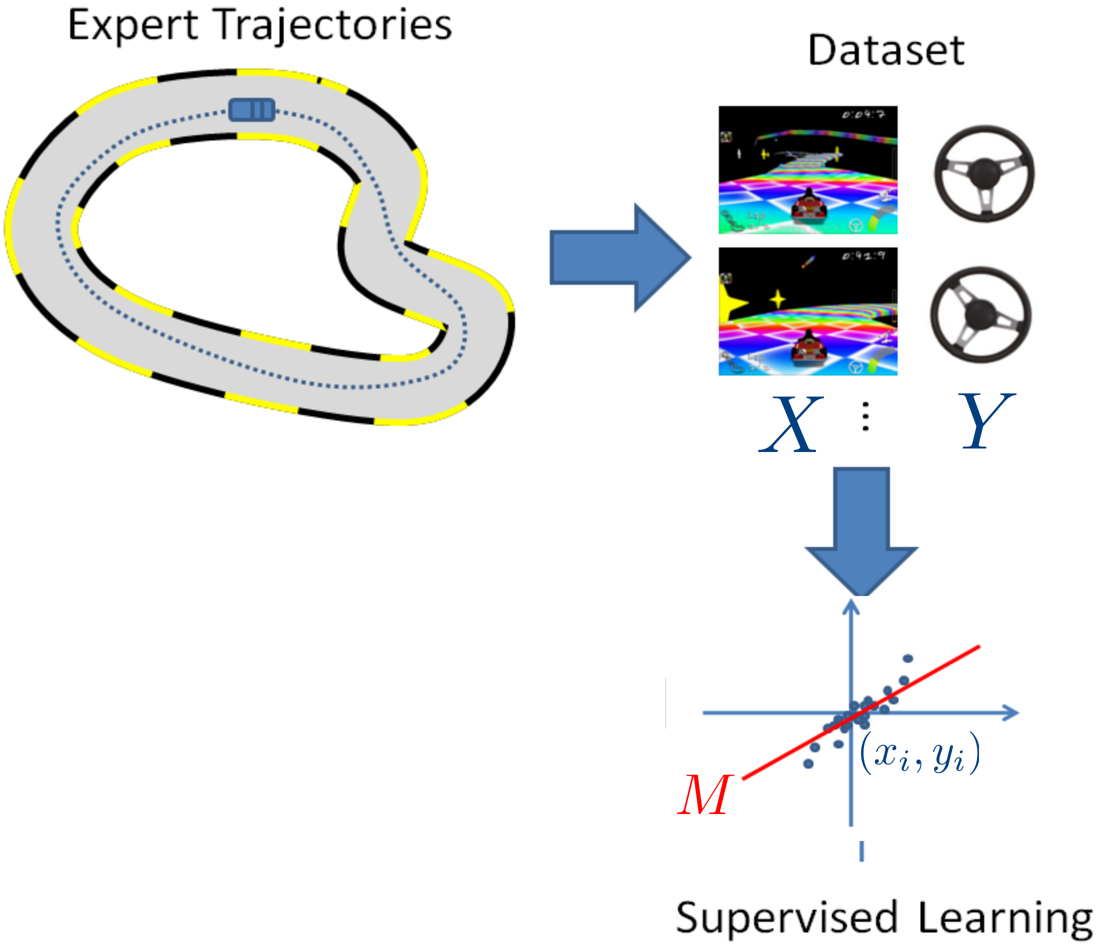

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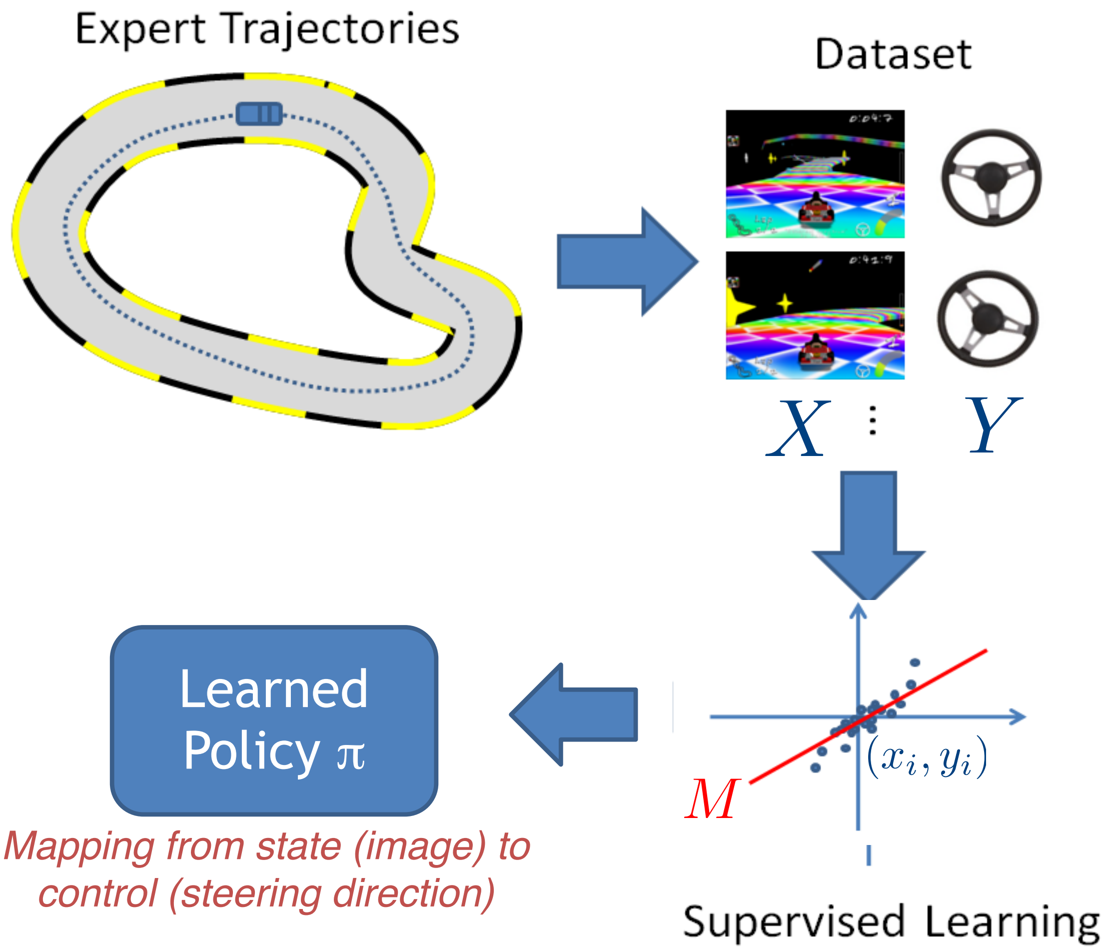

3b. Offline Imitation Learning: Behavior Cloning

Let's formalize the offline IL Setting and the Behavior Cloning algorithm

Discounted infinite horizon MDP $\mathscr{M} = \{S, A, \gamma, r, P, \rho, \pi^\star\}$

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We have a dataset $\mathscr{D} = (s_i^\star, a_i^\star)_{i=1}^M \sim d^{\pi^\star}$

Goal: learn a policy from $\mathscr{D}$ that is as good as the expert $\pi^\star$

Let's formalize the Behavior Cloning algorithm

BC Algorithm input: a restricted policy class $\Pi = \{\pi : S \mapsto \Delta(A)\}$

BC is a Reduction to Supervised Learning:

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$\pi(s_i^*) \approx a_i^*$

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1. Negative log-likelihood (NLL): $\ell(\pi, s, a^\star) = -\ln \pi(a^\star \mid s^\star)$

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2. square loss (i.e., regression for continuous action): $\ell(\pi, s, a^\star) = \|\vec{\pi}(s) - \vec{a}^\star\|_2^2$

$$\widehat{\pi} = \arg\min_{\pi \in \Pi} \sum_{i=1}^{M} \ell\left(\pi, s^\star, a^\star\right)$$

Analysis

Assumption: we are going to assume Supervised Learning succeeded

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Note that here training and testing mismatch at this stage!

Analysis

Theorem [BC Performance] With probability at least $1 - \delta$, BC returns a policy $\hat{\pi}$:

$$V^{\pi^\star} - V^{\hat{\pi}} \leq \frac{2}{(1-\gamma)^2}\epsilon$$

$$\leq 2H^2 \varepsilon$$

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The quadratic amplification is annoying

Summary:

1. TRPO/NPG/PPO
2. Exploration/μ/Guarantees
3. Behavioral Cloning

1-minute feedback form: https://bit.ly/3RHtIxy