From LQR to Nonlinear Control

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CS/Stat 184: Introduction to Reinforcement Learning
Fall 2023

Today

- Feedback from last lecture
- Recap
- Locally linearization
- Iterative LQR

Feedback from feedback forms

1. Thank you to everyone who filled out the forms!
2. Ask class questions

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Recap: LQR

Problem Statement (finite horizon, time homogeneous):

$$\arg \min_{\pi_0,\ldots,\pi_{H-1}:\mathbb{R}^d \to \mathbb{R}^k} \mathbb{E}\left[x_H^\top Q x_H + \sum_{h=0}^{H-1} (x_h^\top Q x_h + u_h^\top R u_h) \right]$$

such that $x_{h+1} = Ax_h + Bu_h + w_h\,, \quad x_0 \sim \mu_0\,, \quad u_h = \pi_h(x_h)\,, \quad w_h \sim N(0,\sigma^2 I)$

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- States $x_h \in \mathbb{R}^d$
- Actions/controls $u_h \in \mathbb{R}^k$
- Additive noise $w_h \sim \mathcal{N}(0,\sigma^2 I)$
- Dynamics linear with state coefficient matrix $A \in \mathbb{R}^{d\times d}$ and action coefficient matrix $B \in \mathbb{R}^{d\times k}$
- Cost function quadratic with positive semidefinite state coefficient matrix $Q \in \mathbb{R}^{d\times d}$ and positive semidefinite action coefficient matrix $R \in \mathbb{R}^{k\times k}$

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$$V_H^\star(x) = x^\top Q x, \text{ define } P_H = Q, p_H = 0,$$

We showed that $V_h^\star(x) = x^\top P_h x + p_h$, where:

$$P_h = Q + A^\top P_{h+1} A - A^\top P_{h+1} B (R + B^\top P_{h+1} B)^{-1} B^\top P_{h+1} A$$

$$p_h = \text{tr}\left(\sigma^2 P_{h+1}\right) + p_{h+1}$$

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Along the way, we also showed that $\pi_h^\star(x) = -K_h x$, where:

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Optimal policy has nothing to do with initial distribution μ_0 or the noise σ^2!

Time-Dependent Costs and Dynamics

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Exact same derivation, only thing that changes is the Ricatti equation:

$$P_h = Q_h + A_h^\top P_{h+1} A_h - A_h^\top P_{h+1} B_h (R_h + B_h^\top P_{h+1} B_h)^{-1} B_h^\top P_{h+1} A_h$$

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Derivation is quite similar, just more algebra!

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Can you see why we already know how to solve this?

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Can you see why we already know how to solve this?

Expanding all the quadratic terms produces a special case of the previous slide!

Beyond LQR

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So far: many extensions to LQR essentially reduce to the same problem

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But what about problems with nonlinear dynamics and/or nonquadratic costs?

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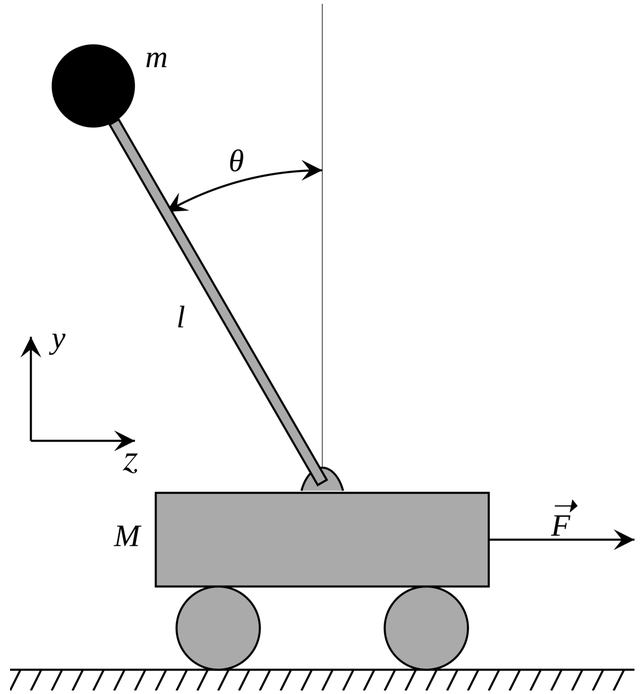
Setting for Local Linearization Approach:



Goal: stabilizing around the goal $(x = x^\star, u = u^\star)$

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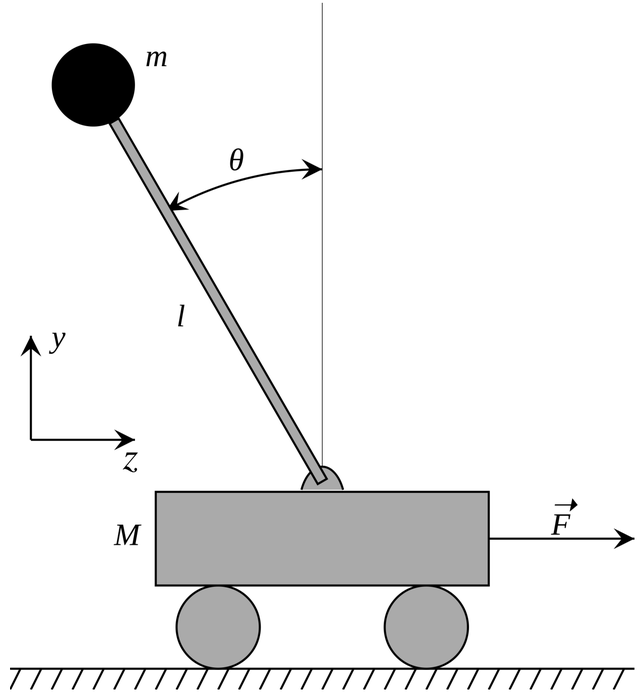

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minimize $\mathbb{E}_\pi\left[\sum_{h=0}^{H-1} c(x_h, u_h)\right]$

s.t. $x_{h+1} = f(x_h, u_h), \quad u_h = \pi(x_h), \quad x_0 \sim \mu_0$

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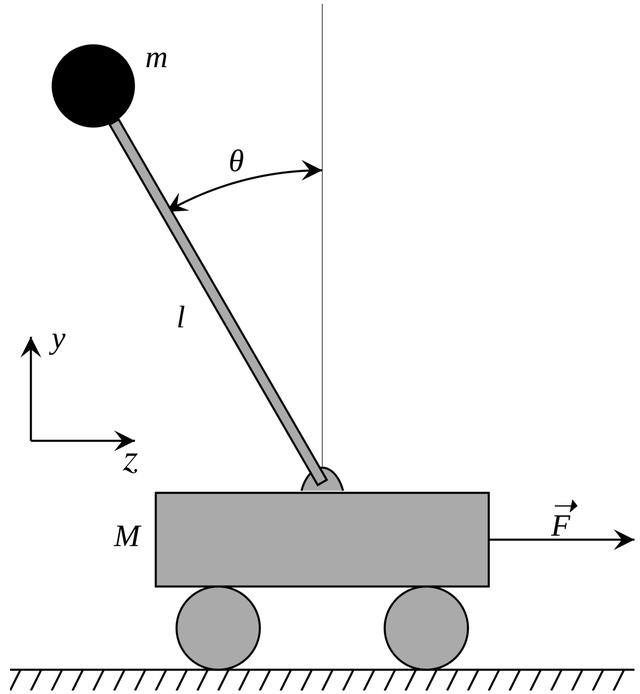

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No noise!

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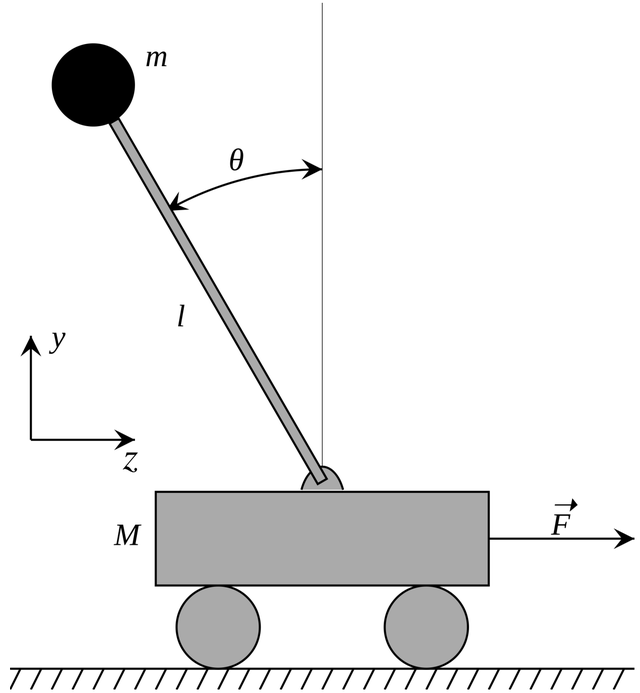

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No noise! No terminal cost $c_h(x_H)$!

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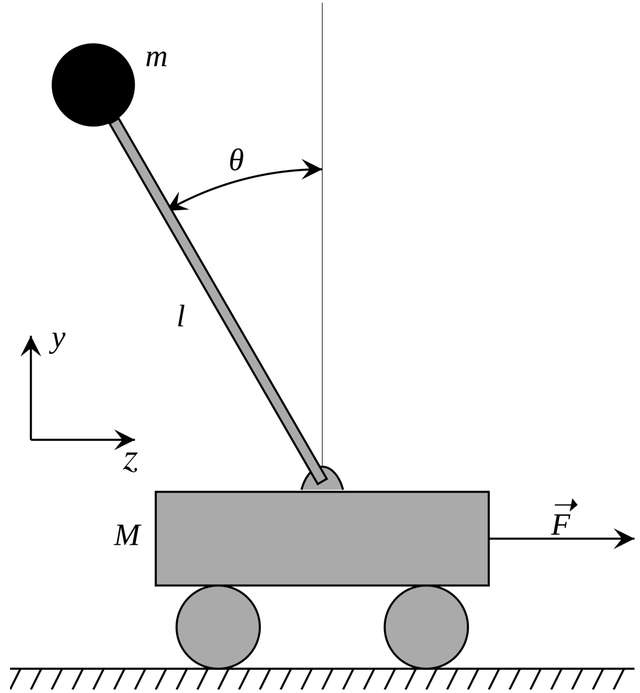

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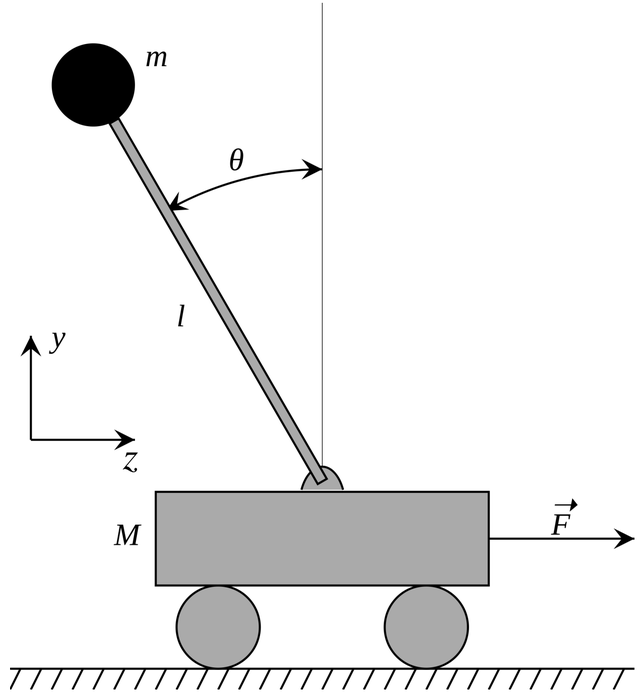

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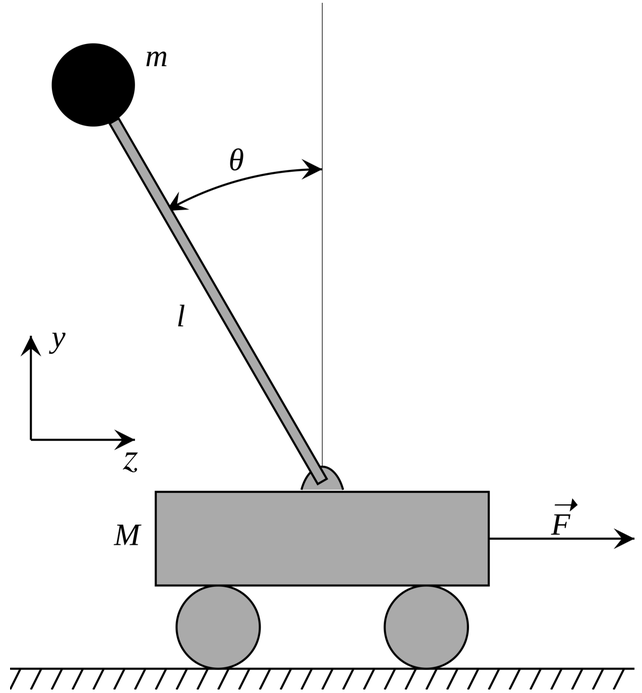

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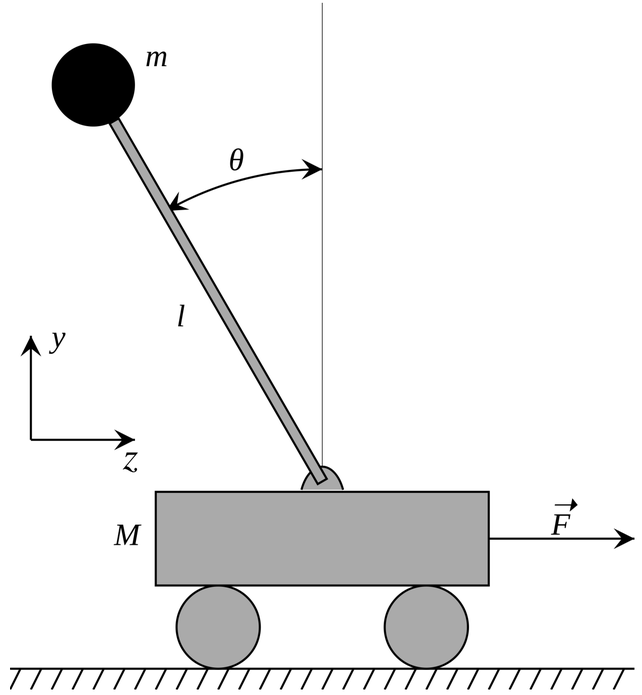

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$\nabla_x f(x,u), \nabla_u f(x,u), \nabla_x c(x,u), \nabla_u c(x,u),$
$\nabla_x^2 c(x,u), \nabla_u^2 c(x,u), \nabla_{x,u}^2 c(x,u)$

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We can approximate $f(x, u)$ locally with a first-order Taylor expansion:

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where:

$$\nabla_x f(x,u) \in \mathbb{R}^{d\times d}, \quad \nabla_x f(x,u)[i,j] = \frac{\partial f[i]}{\partial x[j]}(x,u)$$

$$\nabla_u f(x,u) \in \mathbb{R}^{d\times k}, \quad \nabla_u f(x,u)[i,j] = \frac{\partial f[i]}{\partial u[j]}(x,u)$$

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$$+\frac{1}{2}(x - x^\star)^\top \nabla_x^2 c(x^\star,u^\star)(x - x^\star) + \frac{1}{2}(u - u^\star)^\top \nabla_u^2 c(x^\star,u^\star)(u - u^\star) + (x - x^\star)^\top \nabla_{x,u}^2 c(x,u)(u - u^\star)$$

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$$\nabla_x c(x,u) \in \mathbb{R}^d, \quad \nabla_x c(x,u)[i] = \frac{\partial c}{\partial x[i]}(x,u),$$

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$$\nabla_{x,u}^2 c(x,u) \in \mathbb{R}^{d\times k}, \quad \nabla_{x,u}^2 c(x,u)[i,j] = \frac{\partial^2 c}{\partial x[i]\partial u[j]}(x,u)$$

Local Linearization: Putting it all Together

$$c(x,u) \approx c(x^\star,u^\star) + \nabla_x c(x^\star,u^\star)^\top (x-x^\star) + \nabla_u c(x^\star,u^\star)^\top (u-u^\star)$$
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Rearranging terms, we get back to the following formulation:

$$\arg \min_{\pi_0,\ldots,\pi_{H-1}:\mathbb{R}^d \to \mathbb{R}^k} \mathbb{E}\left[\sum_{h=0}^{H-1} (x_h^\top Q x_h + u_h^\top R u_h + u_h^\top M x_h + x_h^\top q + u_h^\top r + c)\right]$$

such that $x_{h+1} = Ax_h + Bu_h + v\,, \quad x_0 \sim \mu_0\,, \quad u_h = \pi_h(x_h)$

Special case of one of the LQR extensions!

Summary of Local Linearization So Far:

For tasks such as balancing near goal state $(x^\star, u^\star)$,
we can perform **first order Taylor expansion on** $f(x, u)$,
and **second order Taylor expansion on** $c(x, u)$ around the balancing point $(x^\star, u^\star)$

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Last step: checking some practical issues

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So, $\nabla_x^2 c(x^\star, u^\star)$ & $\nabla_u^2 c(x^\star, u^\star)$ may not be positive definite

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What can we do?

In practice, we force them to be positive definite:

Given a *symmetric matrix* $W \in \mathbb{R}^{d \times d}$,

we compute the eigen-decomposition $W = \sum_{i=1}^{d} \sigma_i z_i z_i^\top$, and we approximate W as

$$W \approx \sum_{i=1}^{d} \mathbf{1}(\sigma_i > 0) \sigma_i z_i z_i^\top + \lambda I,$$

for some small $\lambda > 0$

Computing Approximate Derivatives

Recall our assumption: we only have black-box access to f & c:

i.e., unknown analytical form, but given any (x, u), the black boxes output x', c, where

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First implement finite differencing procedure for $\partial c / \partial x[i]$, and then perform another finite differencing with respect to $u[j]$ on top of the first finite differencing procedure for $\partial c / \partial x[i]$

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4. The approximation is an (direct extension of) LQR, so we know how to compute the optimal policy

Today

- ✓ Feedback from last lecture
- ✓ Recap
- ✓ Locally linearization
- Iterative LQR

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But when x_h is far away from $x^\star$ or u_h needs to be far from $u^\star$ for any h,
first/second-order Taylor expansion is not accurate anymore

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$$\text{such that} \quad x_{h+1} = A_h x_h + B_h u_h + v_h\,, \quad x_0 \sim \mu_0\,, \quad u_h = \pi_h(x_h)$$

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Question: how to choose the waypoints $(\bar{x}_h, \bar{u}_h)$ to get the best approximation/solution?

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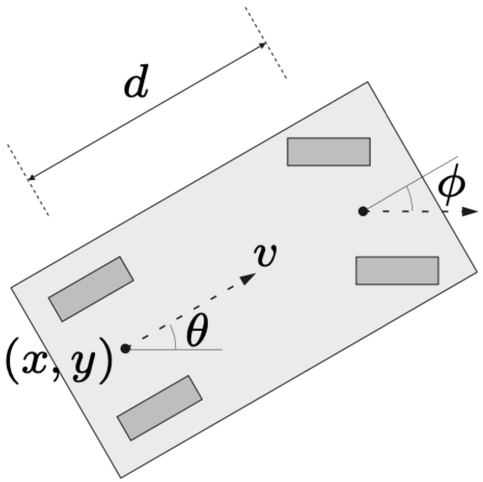
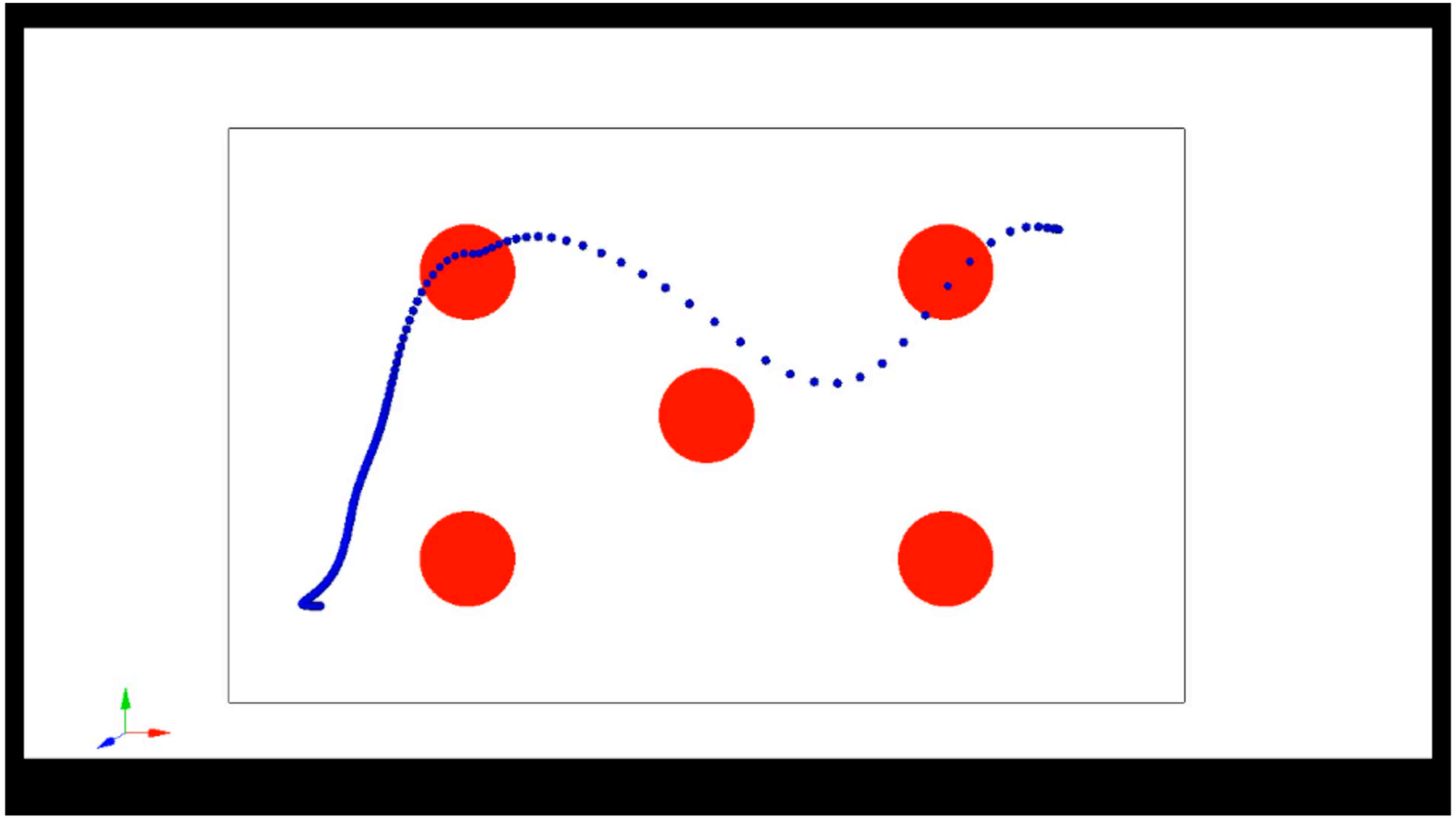
Why is this tractable? because it is 1-dimensional!

Example:

2-d car navigation

Cost function is designed such that it gets to the goal without colliding with obstacles (in red)





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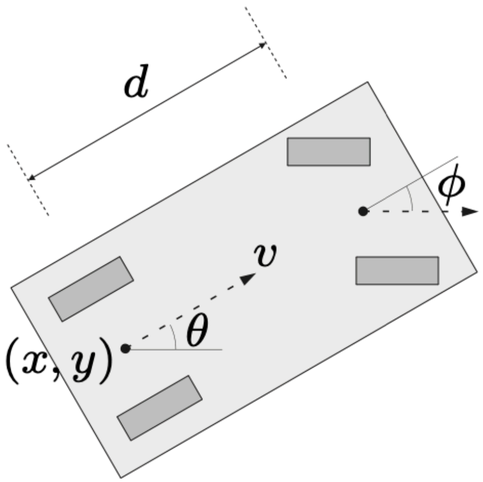

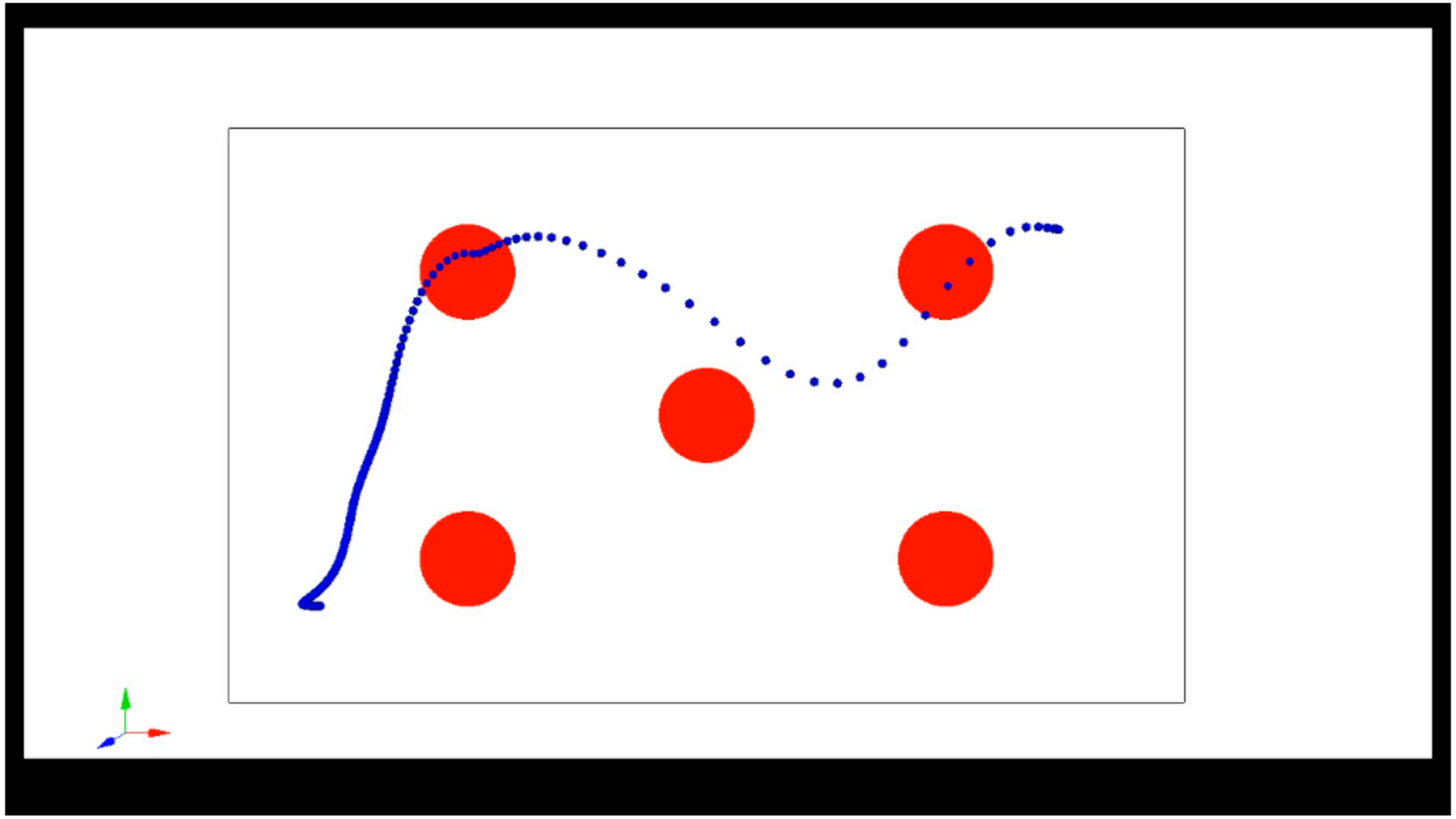

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Today

- ✓ Feedback from last lecture
- ✓ Recap
- ✓ Locally linearization
- ✓ Iterative LQR

Summary:

Local linearization

- Allows us to approximately optimally control any system near its optimum

Iterative LQR

- Uses LQR approximation to find locally optimal nonlinear control solution

Attendance:

bit.ly/3RcTC9T

Feedback:

bit.ly/3RHtlxy