

Contextual Bandits & a Real-world RL Case Study

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CS/Stat 184: Introduction to Reinforcement Learning

Fall 2023

Today

- Contextual Bandits
- LinUCB
- Real world RL example

Contextual bandit environment

Formally, a contextual bandit is the following interactive learning process:

For $t = 0 \rightarrow T - 1$

1. Learner sees context $x_t \sim \nu_x$ Independent of any previous data
2. Learner pulls arm $a_t = \pi_t(x_t) \in \{1, \dots, K\}$ π_t policy learned from all data seen so far
3. Learner observes reward $r_t \sim \nu^{(a_t)}(x_t)$ from arm a_t in context x_t

Note that if the context distribution ν_x always returns the same value (e.g., 0), then the contextual bandit reduces to the original multi-armed bandit

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Not *identical* readership, but still both on NYT, so probably still *similar* readership!

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Lower dimension makes learning easier, but model could be **wrong/biased**

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$$\hat{\theta}_t^{(k)} = \left(\sum_{\tau=0}^{t-1} x_\tau x_\tau^\top \mathbf{1}_{\{a_\tau=k\}} \right)^{-1} \sum_{\tau=0}^{t-1} x_\tau r_\tau \mathbf{1}_{\{a_\tau=k\}}$$

$(X^\top X)^{-1}$ $X^\top Y$

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Apply to $x_t^\top \hat{\theta}_t^{(k)} - x_t^\top \theta^{(k)}$

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Large when $N_t^{(k)}$ small or x_t not aligned with historical data

LinUCB algorithm

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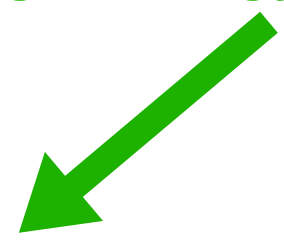
1. $\forall k$, define $A_t^{(k)} = \sum_{\tau=0}^{t-1} x_\tau x_\tau^\top \mathbf{1}_{\{a_\tau=k\}} + \lambda I$ and $\hat{\theta}_t^{(k)} = (A_t^{(k)})^{-1} \sum_{\tau=0}^{t-1} x_\tau r_\tau \mathbf{1}_{\{a_\tau=k\}}$

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i. $1/\delta$ (δ is probability you want the bound to hold with)

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Can prove $\tilde{O}(\sqrt{T})$ regret bound

Today

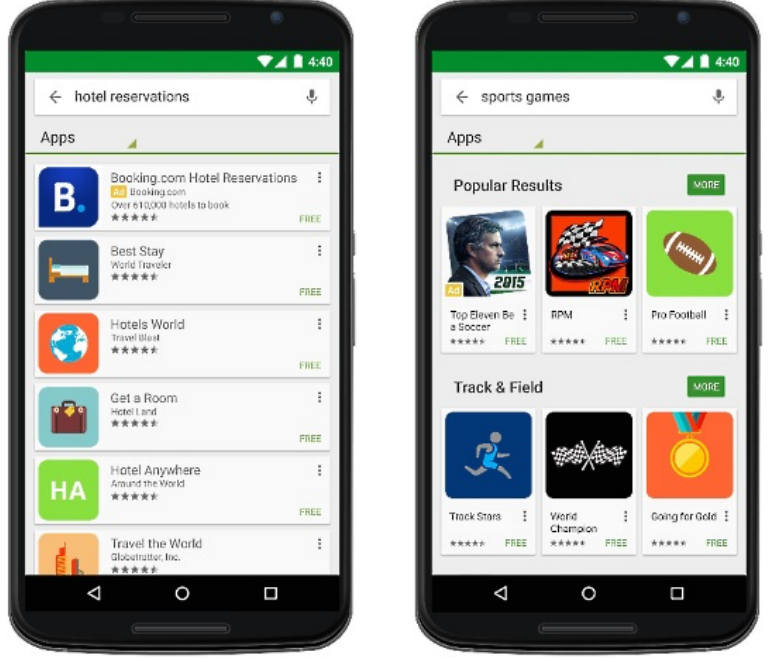
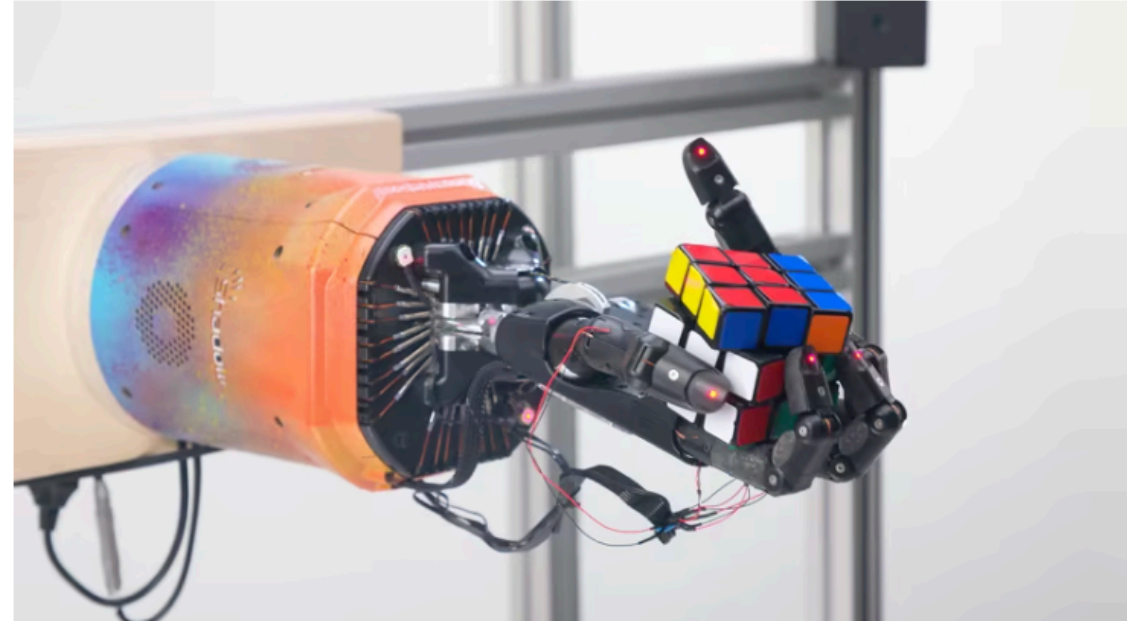
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Case Study: RL for Supply Chains

Real-world RL is hard.

Many RL successes in controlled domains.

How can RL add value in the real world?



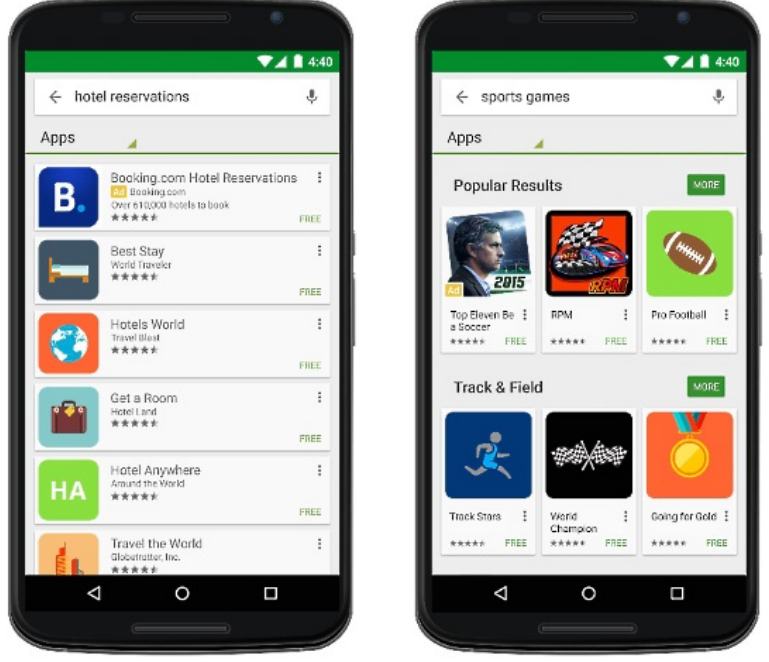
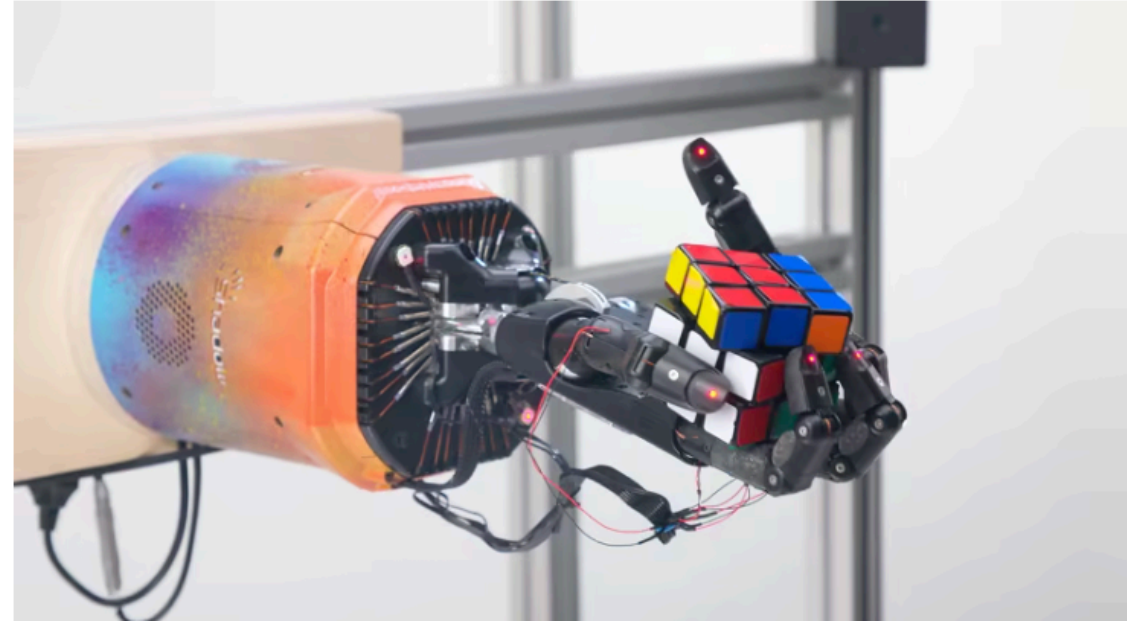
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Issues:
sample complexity?
how to use offline data?
exploration/counterfactual reasoning?



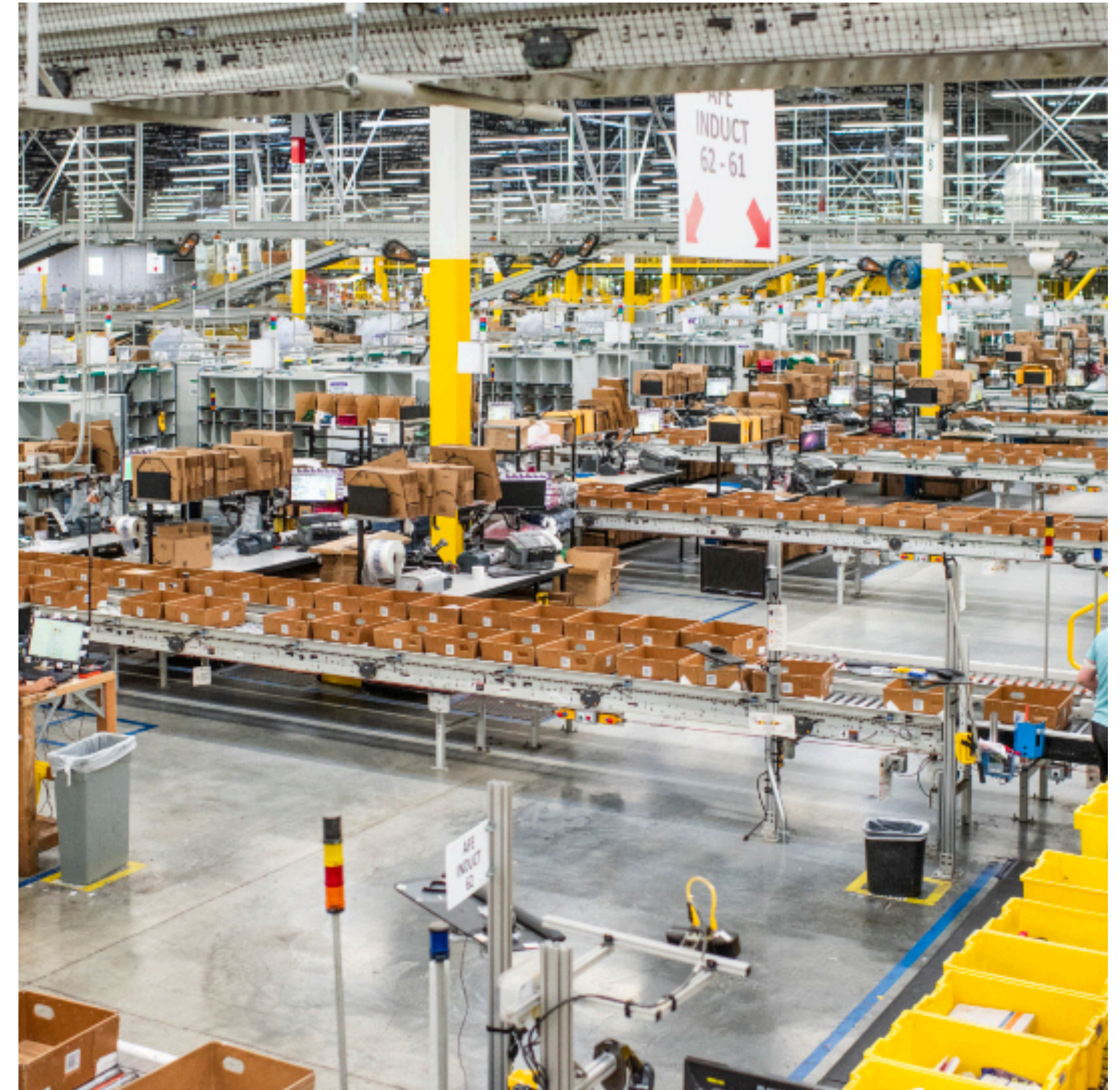
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- Supply Chain is about buying, storing, and transporting goods.
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- **Today:** how can we **use this data** to solve the inventory management problem?
 - counterfactual issues?

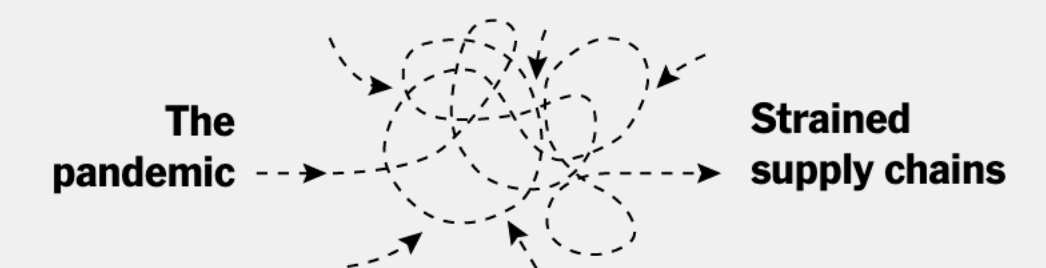


Supply Chain Hurdles Will Outlast Pandemic, White House Says

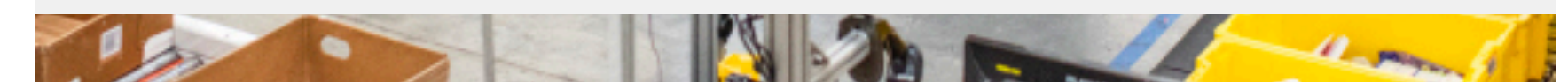
The administration's economic advisers see climate change and other factors complicating global trade patterns for years to come.



The New York Times



How the Supply Chain Crisis Unfolded



Outline

Can we use historical data to solve inventory management problems in supply chain?

- How to use historical data?
- Moving to real-world inventory management problems
- Real world results

Largely based on this paper:
[arxiv/2210.03137](https://arxiv.org/abs/2210.03137)

Deep Inventory Management

Dhruv Madeka
Amazon, maded@amazon.com

Kari Torkkola
Amazon, karito@amazon.com

Carson Eisenach
Amazon, ceisen@amazon.com

Anna Luo
Pinterest*, annaluo676@gmail.com

Dean P. Foster
Amazon, foster@amazon.com

Sham M. Kakade
Amazon, Harvard University, shamisme@amazon.com

I: Utilizing historical data

Warm up: Vehicle Routing

(when using historical data might be ok)

Paul G. Allen Center for Computer Scienc

CorePower Yoga, 300 3rd Ave W, Seattle,

Add destination

Leave now Options

Send directions to your phone

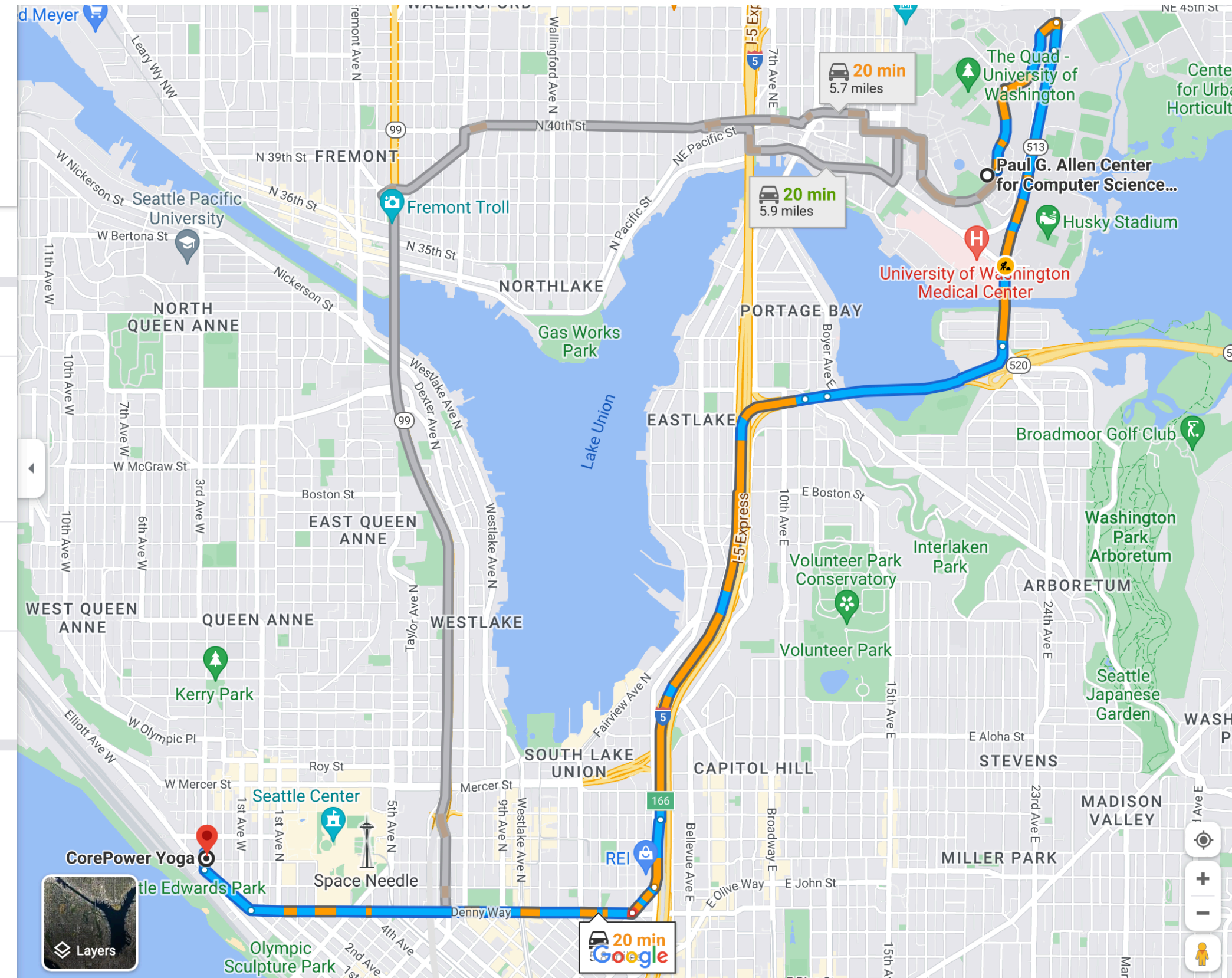
via Denny Way **20 min**
Fastest route now due to traffic conditions
5.8 miles
[Details](#)

via Aurora Ave N **20 min**
Lighter traffic than usual
5.7 miles

via NE 40th St and Aurora Ave N **20 min**
5.9 miles

Explore CorePower Yoga

Restaurants Hotels Gas stations Parking Lots More



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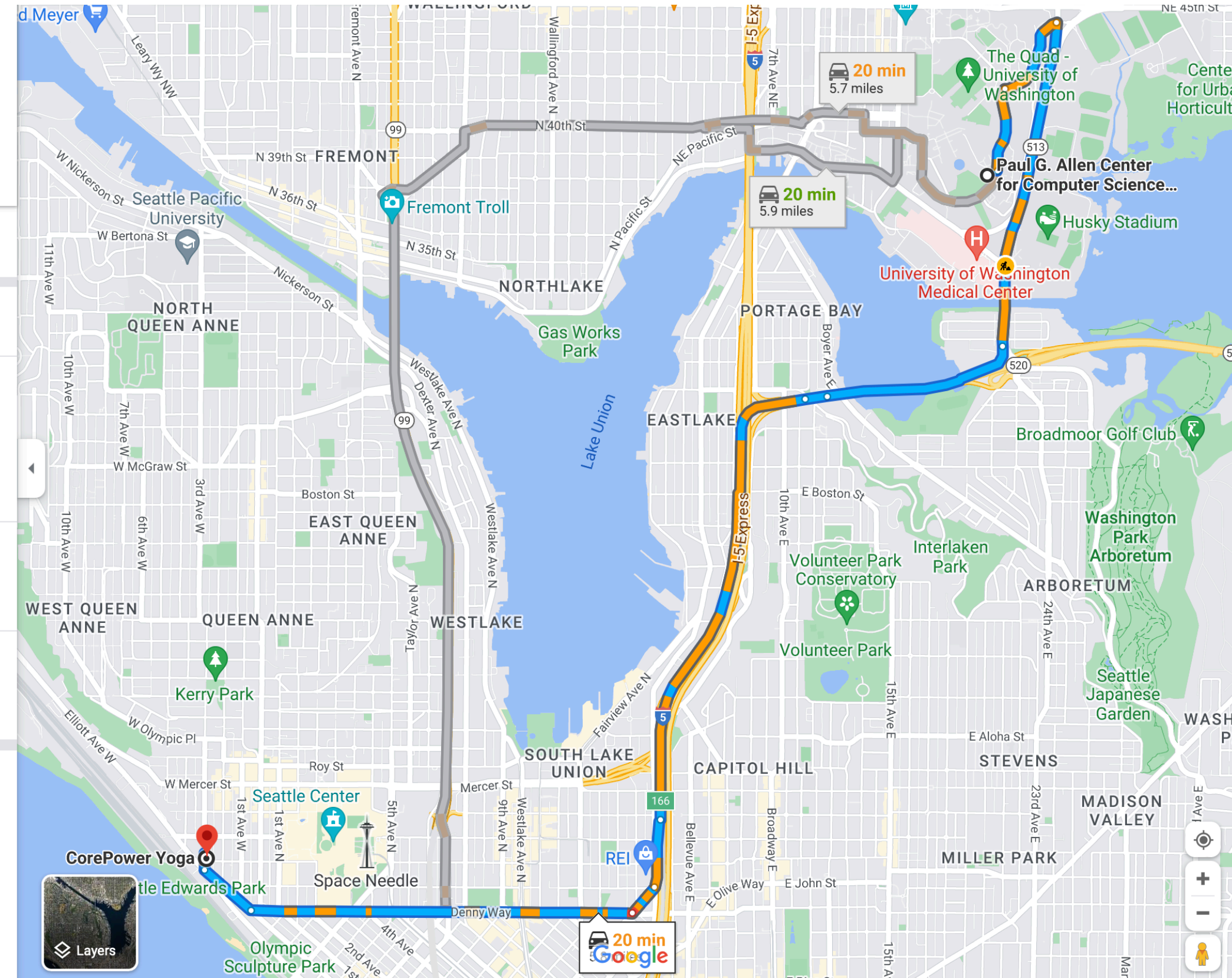
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- **Policy π : features \rightarrow directions**
features: time of day, holiday indicators, current traffic, sports games, accidents, location, weather,

The screenshot displays a Google Maps interface with the following elements:

- Search Bar:** Contains the destination "CorePower Yoga, 300 3rd Ave W, Seattle,".
- Destination List:** Shows "Paul G. Allen Center for Computer Scienc" as the starting point and "CorePower Yoga, 300 3rd Ave W, Seattle," as the destination.
- Route Options:** Lists three routes:
 - via Denny Way:** 20 min, 5.8 miles. Description: "Fastest route now due to traffic conditions".
 - via Aurora Ave N:** 20 min, 5.7 miles. Description: "Lighter traffic than usual".
 - via NE 40th St and Aurora Ave N:** 20 min, 5.9 miles.
- Map:** Shows a map of Seattle with a highlighted route in blue and orange. Key landmarks include Lake Union, Fremont Troll, University of Washington Medical Center, and the Paul G. Allen Center for Computer Science.
- Bottom Bar:** Includes icons for "Restaurants", "Hotels", "Gas stations", "Parking Lots", and "More".

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- **Historical Data:**
suppose we have logged historical data of features

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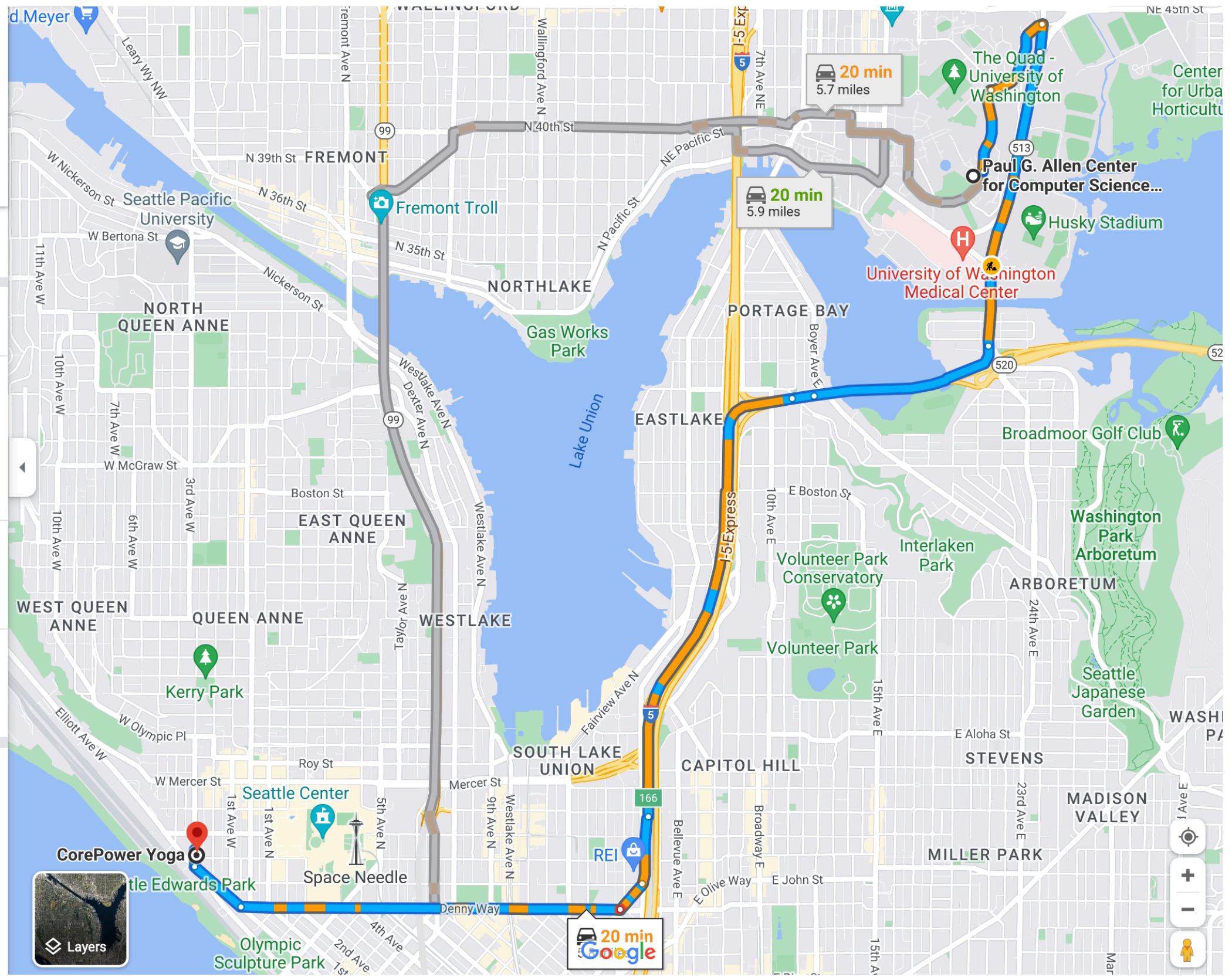
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via NE 40th St and Aurora Ave N	20 min	5.9 miles

Explore CorePower Yoga

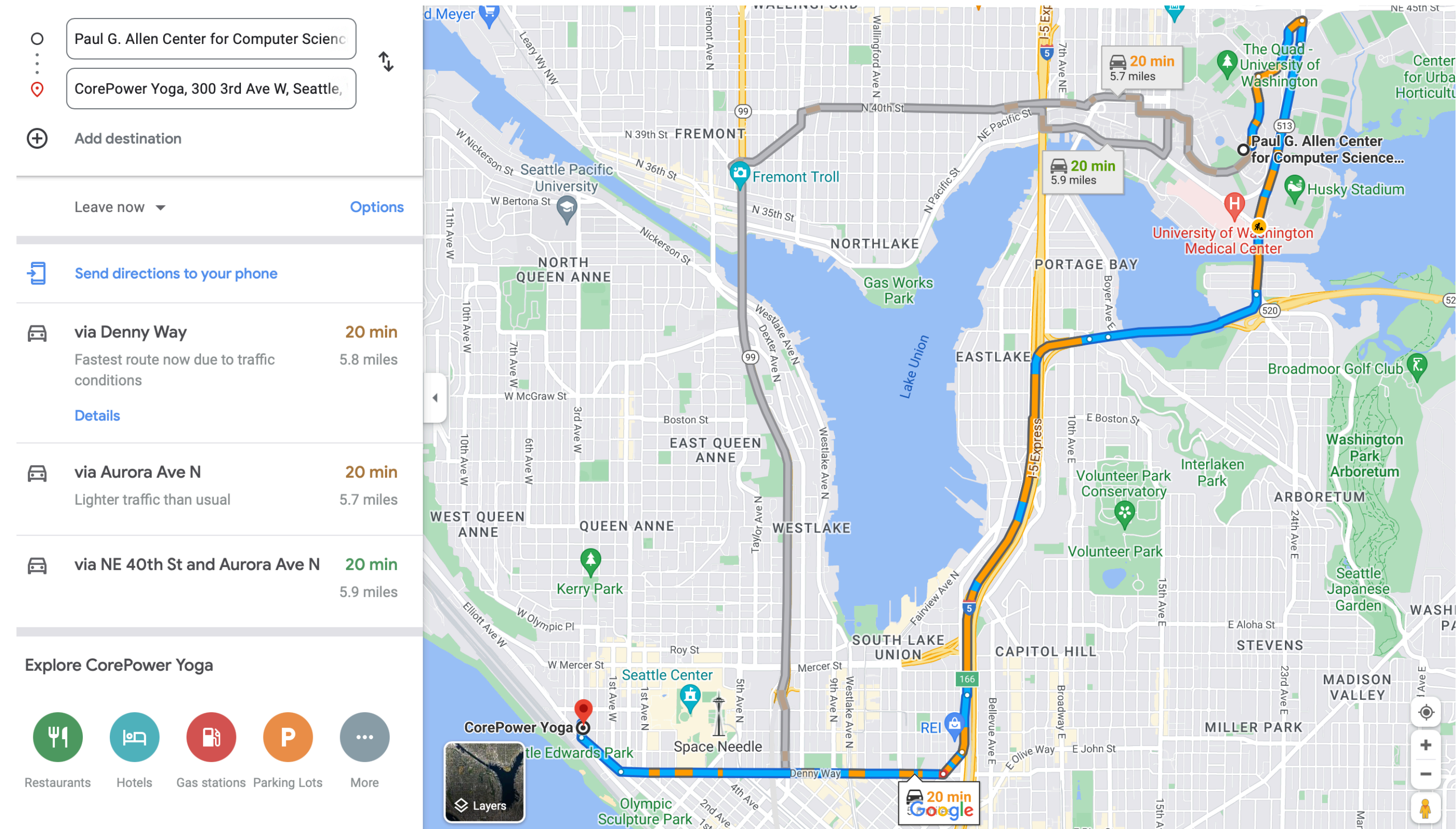
Restaurants Hotels Gas stations Parking Lots More



Warm up: Vehicle Routing

(when using historical data might be ok)

- We want a good policy for routing a single car.
- **Policy π : features \rightarrow directions**
features: time of day, holiday indicators, current traffic, sports games, accidents, location, weather,
- **Historical Data:**
suppose we have logged historical data of features
- **Backtesting policies:**
 - Key idea: a single route minimally affects traffic
 - **Counterfactual:** with the historical data, we can see what would have happened with another policy.



Warm up 2: Fleet Routing



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- We want to route a whole fleet of self-driving taxis.



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- Historical Data:
suppose we have logged historical data of features
- Backtesting policies:
 - Key idea: a small fleet route may have small affects on traffic.
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Supply Chain Data

Supply Chain Data

Price= \$2
Cost= \$1

Time	Inventory	Demand	Order	Revenue

Supply Chain Data

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Time	Inventory	Demand	Order	Revenue
0	100	20	-	40

Supply Chain Data

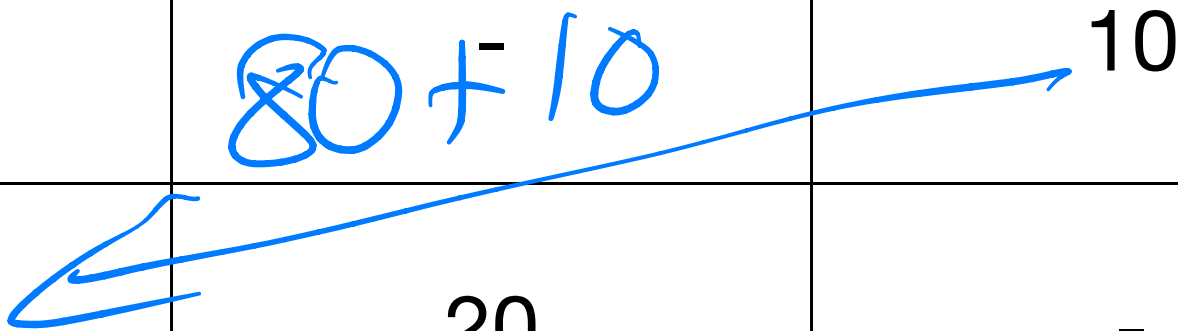
Price= \$2
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Time	Inventory	Demand	Order	Revenue
0	100	20	-	40
0	80	-	10	-10

Supply Chain Data

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Time	Inventory	Demand	Order	Revenue
0	100	20	-	40
0	80	20	10	-10
1	90	20	-	40



Supply Chain Data

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Time	Inventory	Demand	Order	Revenue
0	100	20	-	40
0	80	-	10	-10
1	90	20	-	40
1	70	-	50	-50

Handwritten blue annotations: "90 - 20" with a circle around "20" and an arrow pointing from the "90" in the Inventory column of the third row to the "70" in the Inventory column of the fourth row.

Supply Chain Data

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Time	Inventory	Demand	Order	Revenue
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2	120	60	-	120

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Backtesting a policy

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- Current order doesn't impact future demand.
 - This allows us to backtest!
 - Empirically, backlog due to unmet demand does not look significant.¹

1. See Verhoef et al (2006)

Formalization of the Supply Chain Problem

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- **Exogenous MDPs:** Growing literature around a class of MDPs where a large part of the state is driven by an exogenous noise process [Efroni et al 2021, Sinclair et al 2022]

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 - **Known controllable part (inventory) I_t :** (known) evolution is dependent on our action.
 - $I_t = \max(I_{t-1} + a_{t-1} - D_t, 0)$ (and suppose we start at I_0).
 - Immediate reward is the profits: $r(s_t, I_t, a_t) := \text{Price}_t \times \min(\text{Demand}_t, I_t) - \text{Cost}_t \times a_t$
- $r \mid \text{state} := (I_t, s_t)$

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$$V_H(\pi) = E_{\pi} \left[\sum_{t=1}^H \gamma^t r(s_t, I_t, a_t) \right]$$

Why is it an interesting RL problem?

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- Lots of time dependence!
 - If you buy too much, you're left with the inventory for months!
 - Your actions (orders) affect the state at a random time later
 - Tons of correlation across time (Demand, Price, Cost, Seasonality, etc)

What do ExoMDPs buy us?

We can backtest (assuming the “controllable” dynamics are known) and avoid the counterfactual/causality issue!

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Theorem: RL in ExoMDPs is as easy as Supervised Learning

Suppose we have K policies $\Pi = \{\pi_1, \dots, \pi_K\}$, and we have N sampled exogenous paths. Then we can accurately backtest up to nearly $K \approx 2^N$ policies.

Formally, for $\delta \in (0, 1)$, with pr. greater than $1 - \delta$ - we have that for all $\pi \in \Pi$:

$$|V_0(\pi) - \hat{V}_0(\pi)| \leq H \sqrt{\frac{\log(K/\delta)}{N}}$$

(assuming the reward r_t is bounded by 1).

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- **Implications:**
 - We can optimize a **neural policy** on the past data.
 - In the usual RL setting (not exogenous), we would have an **amplification factor of (at least) $\min\{2^H, K\}$** , using historical data due to the counterfactual issue.

II: Real World Inventory Management Problems

Real-world Issue: **Censored** Demand

- When **demand** \geq **inventory**, what customers see:

Real-world Issue: **Censored** Demand

- When **demand \geq inventory**, what customers see:


\$19.99
& **FREE Shipping**
Get it Tue, Jan 29 - Thu, Jan 31,
or
Get it Fri, Jan 25 - Fri, Jan 25 if
you choose paid **Local Express**
Shipping at checkout

**In stock on January 23,
2019.**

Order it now.
Ships from and sold by Vertellis.

Qty:

\$19.99 + Free Shipping

 **Add to Cart**

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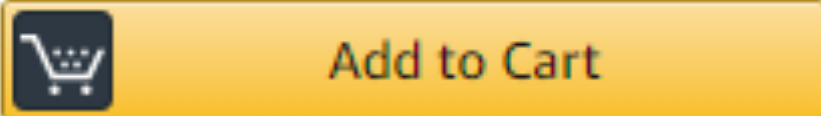
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Buy New **\$18.96**
Qty: 1 ▾ List Price:
\$29.99
Save: \$11.03 (37%)

FREE Shipping on orders over \$35.

Temporarily out of stock.
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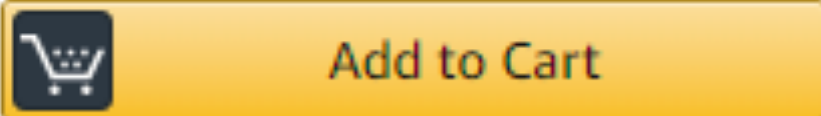
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We only observe **sales** not the **demand**:
Sales := min(Demand, Inventory)

Can we still backtest?

Our historical data is then censored....

Sales := min(Demand, Inventory)

Price= \$2

Cost= \$1

Our historical data is then censored....

$$\text{Sales} := \min(\text{Demand}, \text{Inventory})$$

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Time	Inventory	True Demand	Sales	Order	Revenue
T	10	??	10	-	20
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
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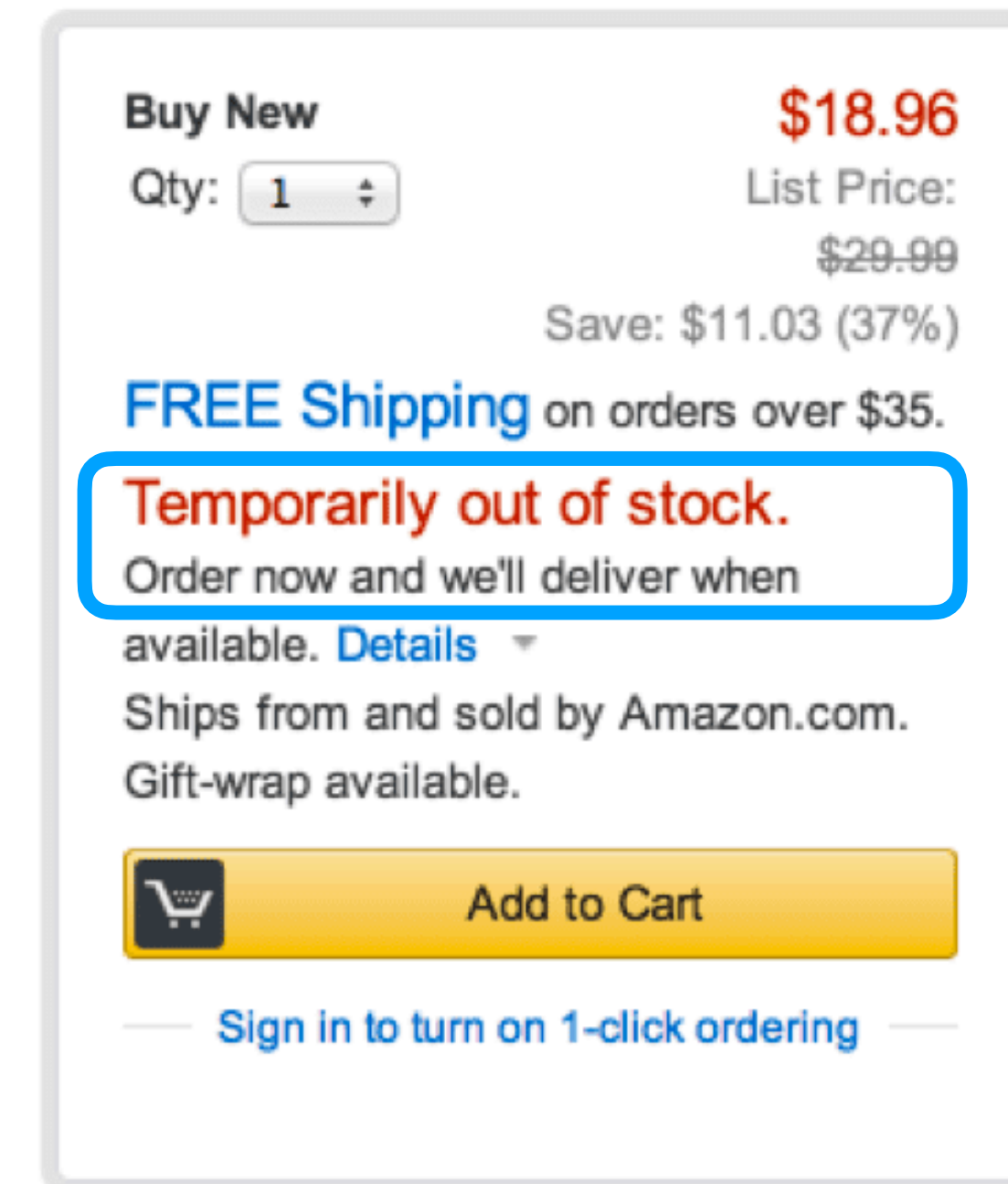
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The image shows two screenshots of an Amazon product page. The left screenshot displays a product priced at \$19.99 with free shipping. A blue box highlights the text "In stock on January 23, 2019." The right screenshot shows the same product but with a blue box highlighting the text "Temporarily out of stock. Order now and we'll deliver when available." This illustrates how the same product's availability can change over time, leading to censored data in historical records.

If we could fill in the missing demand, then we could still backtest!

We have many observed historical covariates

- **Covariates:** Sales, Web Site, **Glance Views**, Product Text, Reviews
- **Example:** the #times customers look at an item gives us info about the unobserved demand.
- **Let's forecast the missing variables** from the observed covariates!
 $\hat{P}(\text{Missing Data} \mid \text{Observed Data})$

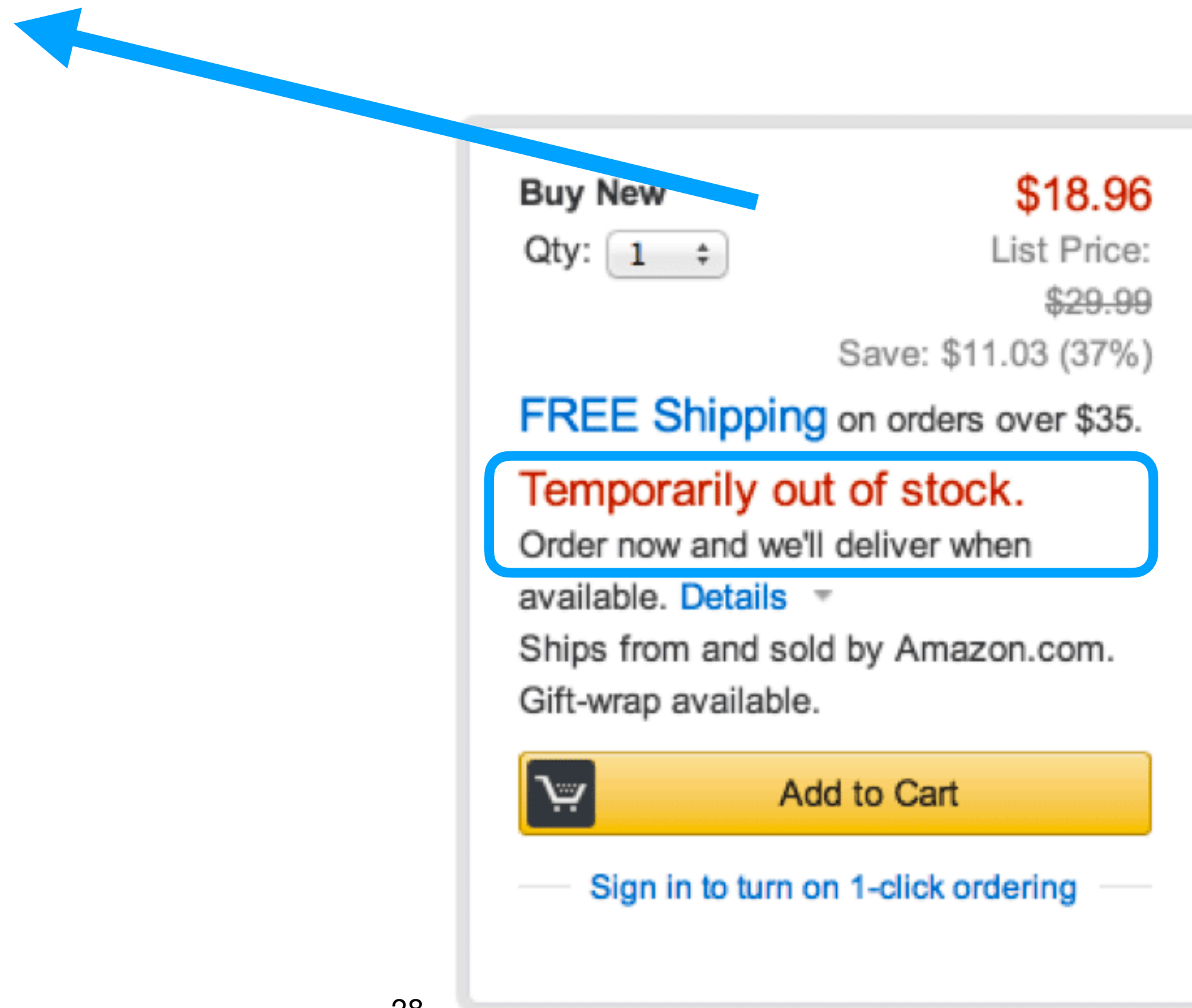


Uncensoring the data....

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Uncensoring the data....

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⋮	⋮	⋮	⋮		
⋮	⋮	⋮	⋮		
⋮	⋮	⋮	⋮		
⋮	⋮	⋮	⋮		
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⋮	⋮	⋮	⋮		

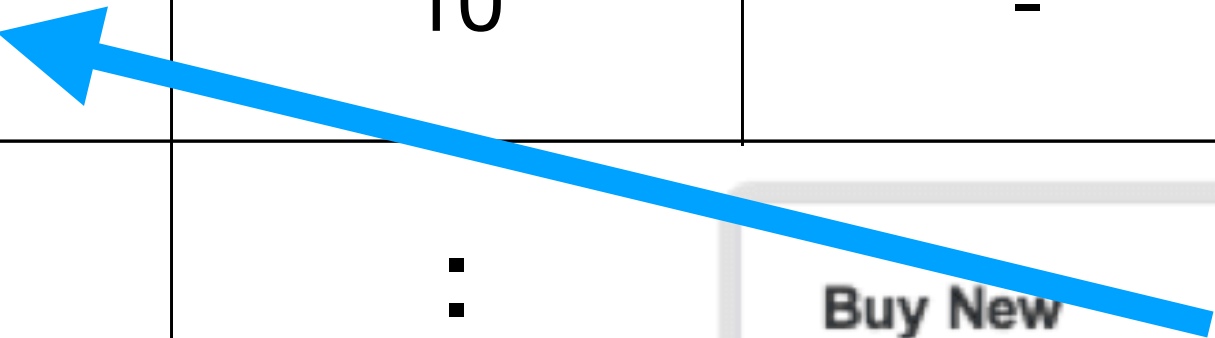
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Key idea:
 Use covariates
 (e.g. glance
 views) to forecast
 missing demand,
 vendor lead
 times, etc

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We can backtest (even with censored data) and avoid the counterfactual/causality issue!

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(with only additive error increase based on our SL error).

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Setting: we have N sampled sequences $\{s_1^i, s_2^i, \dots, s_H^i\}_{i=1}^N$,
where M_i and O_i are the missing and observed exogenous variables in sequence i .

Forecast: $\widehat{\mathbb{P}}^i = \widehat{\Pr}(M_i | O_i)$ is our forecast of $\mathbb{P}^i = \Pr(M_i | O_i)$.

Assume: With pr. 1, forecasting has low error:
$$\frac{1}{N} \sum_{i=1}^N \text{TotalVar}(\mathbb{P}^i, \widehat{\mathbb{P}}^i) \leq \epsilon_{\text{sup}}.$$

Guarantee: For any $\delta \in (0,1)$, with pr. greater than $1 - \delta$, for all $\pi \in \Pi$:

$$|V_0(\pi) - \widehat{V}_0(\pi)| \leq H \left(\epsilon_{\text{sup}} + \sqrt{\frac{\log(K/\delta)}{N}} \right)$$

III: Training Policies & Empirical Results

The Simulator

The Simulator

- Collection of historical trajectories:
 - 1 million products
 - 104 weeks of data per product



The Simulator

- **Collection of historical trajectories:**
 - 1 million products
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- **Uncensoring:**
 - Demand
 - Vendor Lead Times



The Simulator

- **Collection of historical trajectories:**
 - 1 million products
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- **Uncensoring:**
 - Demand
 - Vendor Lead Times
- **Policy gradient methods in a “gym”:**
 - “gym” ↔ backtesting ↔ simulator
(note the “simulator” isn’t a good world model).
 - The policy can depend on many features.
(seasonality, holiday indicators, demand history, product details, text features)

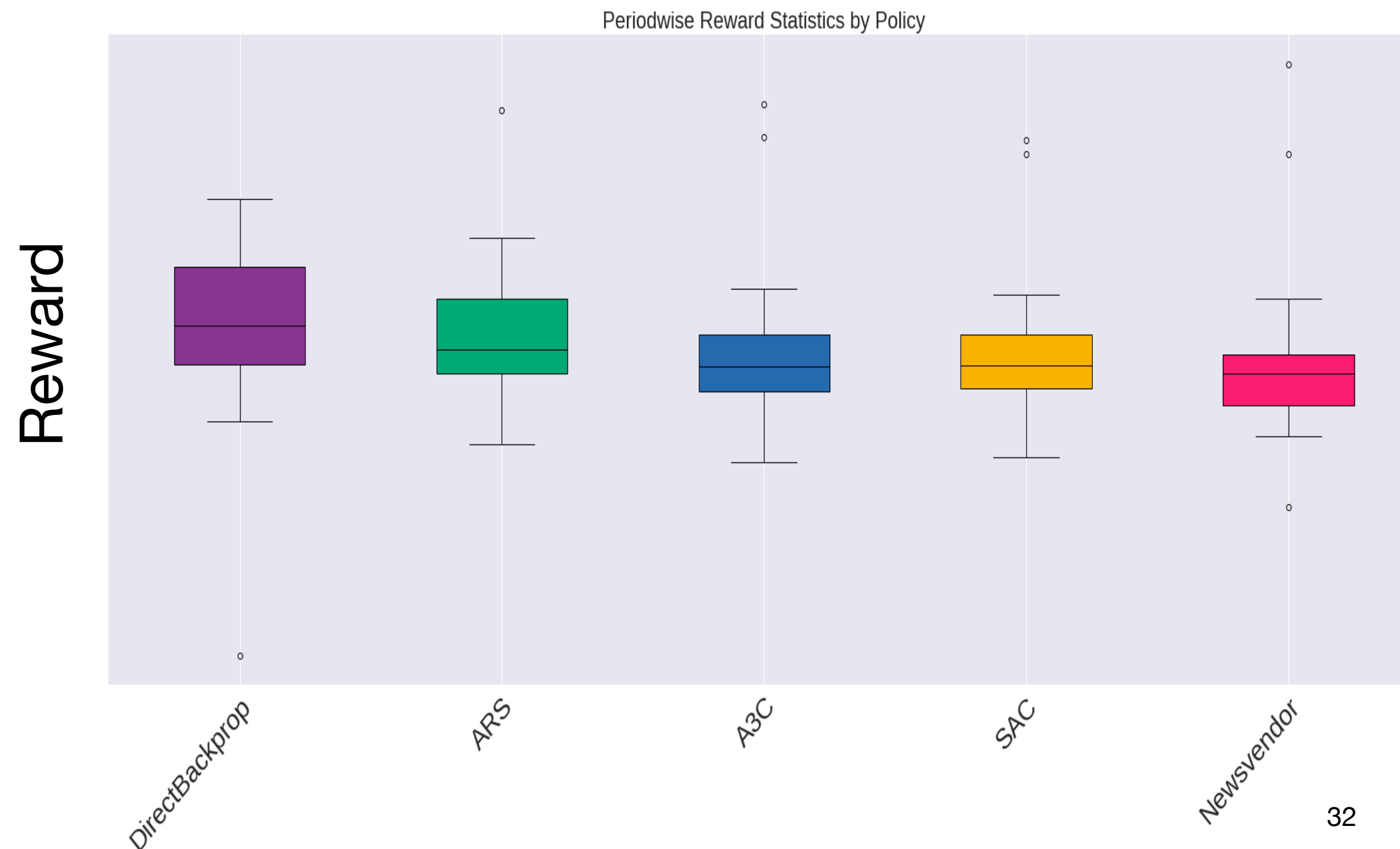


Sim to Real Transfer

- Sim: the backtest of [DirectBackprop](#) improves on Newsvendor.
- Real: [DirectBackprop](#) significantly reduces inventory without significantly reducing total revenue.

Simulation

Real World



Metrics	% change
Inventory Level	-12 ± 6
Revenue	2.6%

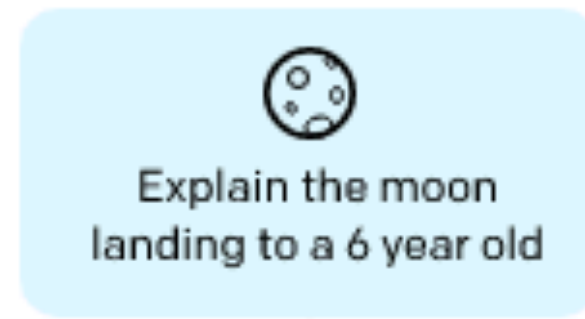
RLHF

RL from Human Feedback (RLHF)

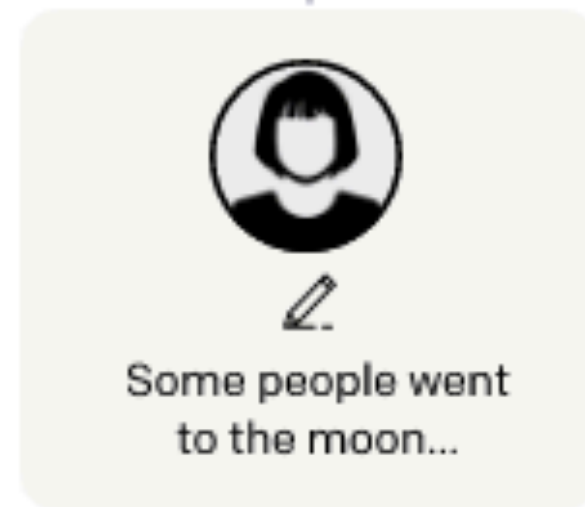
Step 1

Collect demonstration data, and train a supervised policy.

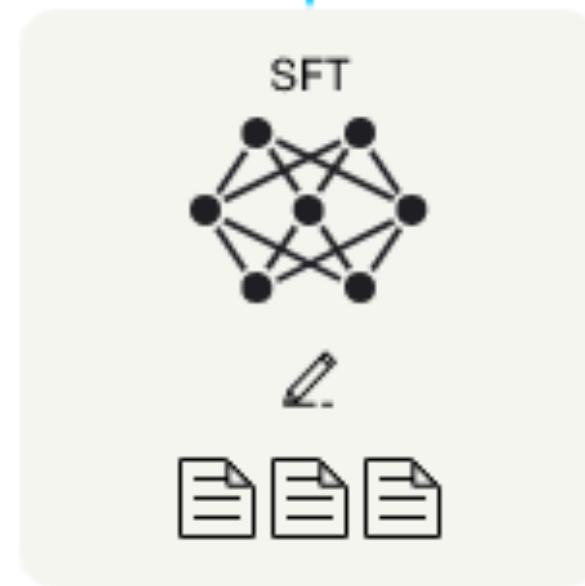
A prompt is sampled from our prompt dataset.



A labeler demonstrates the desired output behavior.



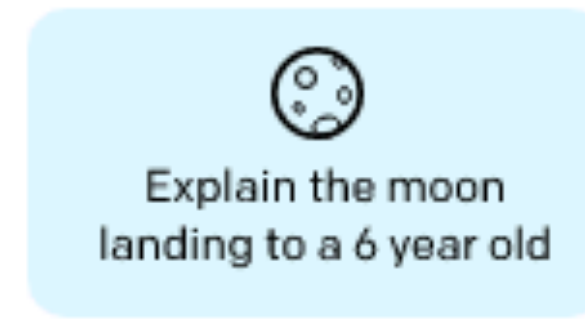
This data is used to fine-tune GPT-3 with supervised learning.



Step 2

Collect comparison data, and train a reward model.

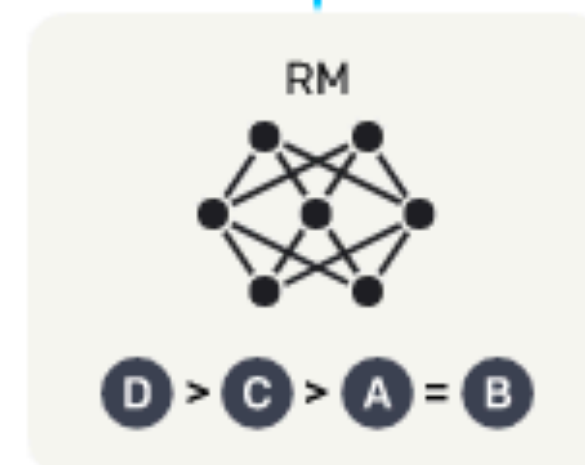
A prompt and several model outputs are sampled.



A labeler ranks the outputs from best to worst.



This data is used to train our reward model.



Step 3

Optimize a policy against the reward model using reinforcement learning.

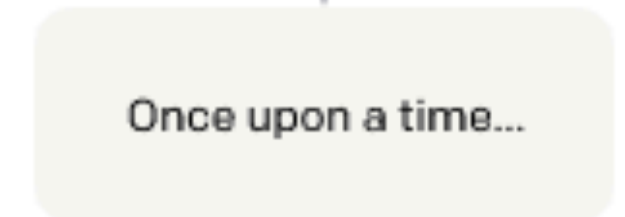
A new prompt is sampled from the dataset.



The policy generates an output.



The reward model calculates a reward for the output.



The reward is used to update the policy using PPO.



Summary:

Attendance:

bit.ly/3RcTC9T



Feedback:

bit.ly/3RHtlxy



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Today: adding context to bandits requires SL but makes it much more useful

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 - RL gives a helpful set of tools.
 - RL also gives an interesting viewpoint.

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- **We hope you enjoyed the course!**

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Both cases allow a version of linUCB by extension of the same ideas: fit coefficients via least squares and use Chebyshev-like uncertainty quantification to get UCB

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- iii. The other formulation, with separate $A_t^{(k)}$ and $\hat{\theta}_t^{(k)}$, is called **disjointed**

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But in principle, there is **no “free lunch”**, i.e., the hardness of the problem now transfers over to choosing a good model (a bad model will lead to bad performance)