Contextual Bandits & a Real-world RL Case Study

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CS/Stat 184: Introduction to Reinforcement Learning
Fall 2023

Today

- Contextual Bandits
- LinUCB
- Real world RL example

Contextual bandit environment

Formally, a contextual bandit is the following interactive learning process:

For $t = 0 \to T - 1$

1. Learner sees context $x_t \sim \nu_x$ — Independent of any previous data
2. Learner pulls arm $a_t = \pi_t(x_t) \in \{1, \ldots, K\}$ — π_t policy learned from all data seen so far
3. Learner observes reward $r_t \sim \nu^{(a_t)}(x_t)$ from arm a_t in context x_t

Note that if the context distribution ν_x always returns the same value (e.g., 0), then the contextual bandit reduces to the original multi-armed bandit

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Example: showing an ad on a NYT article on politics vs a NYT article on sports: Not *identical* readership, but still both on NYT, so probably still *similar* readership!

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Lower dimension makes learning easier, but model could be wrong/biased

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$$\hat{\theta}_t^{(k)} = \left(\sum_{\tau=0}^{t-1} x_\tau x_\tau^\top 1_{\{a_\tau=k\}} \right)^{-1} \sum_{\tau=0}^{t-1} x_\tau r_\tau 1_{\{a_\tau=k\}}$$

$(X^\top X)^{-1}$ $X^\top Y$

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Apply to $x_t^\top \hat{\theta}_t^{(k)} - x_t^\top \theta^{(k)}$

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Large when $N_t^{(k)}$ small or x_t not aligned with historical data

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1. $\forall k$, define $A_t^{(k)} = \sum_{\tau=0}^{t-1} x_\tau x_\tau^\top 1_{\{a_\tau = k\}} + \lambda I$ and $\hat{\theta}_t^{(k)} = (A_t^{(k)})^{-1} \sum_{\tau=0}^{t-1} x_\tau r_\tau 1_{\{a_\tau = k\}}$

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2. Observe context x_t and choose $a_t = \arg\max_k \left\{ x_t^\top \hat{\theta}_t^{(k)} + c_t \sqrt{x_t^\top (A_t^{(k)})^{-1} x_t} \right\}$

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c_t similar to log term in (non-lin)UCB, in that it depends logarithmically on

i. $1/\delta$ (δ is probability you want the bound to hold with)

ii. t and d implicitly via $\det(A_t^{(k)})$

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Can prove $\tilde{O}(\sqrt{T})$ regret bound

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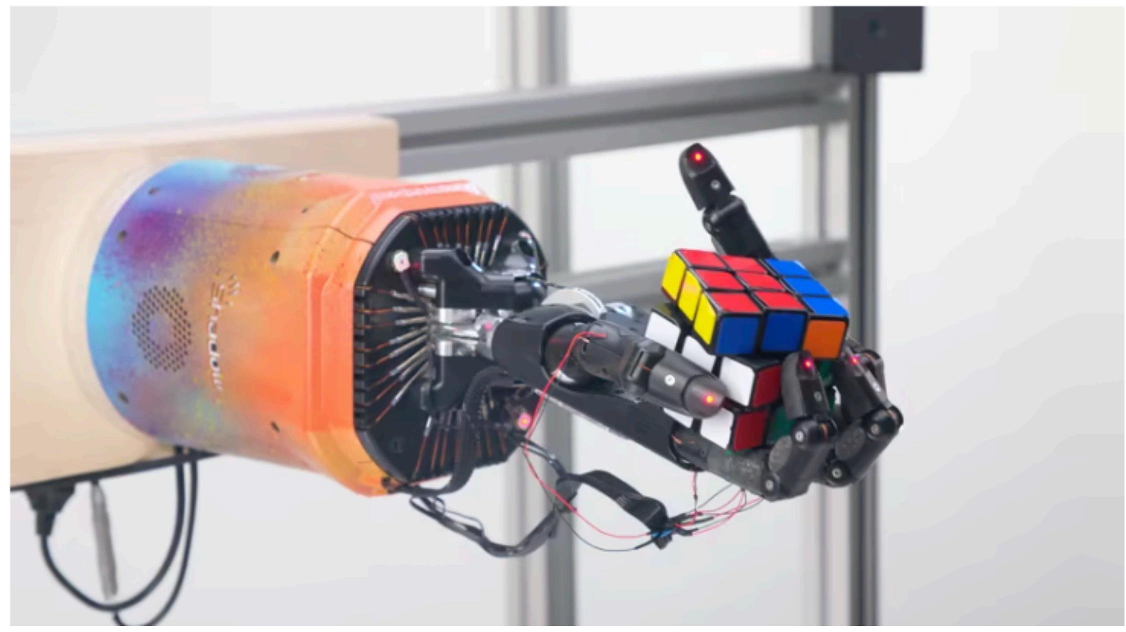
Case Study: RL for Supply Chains

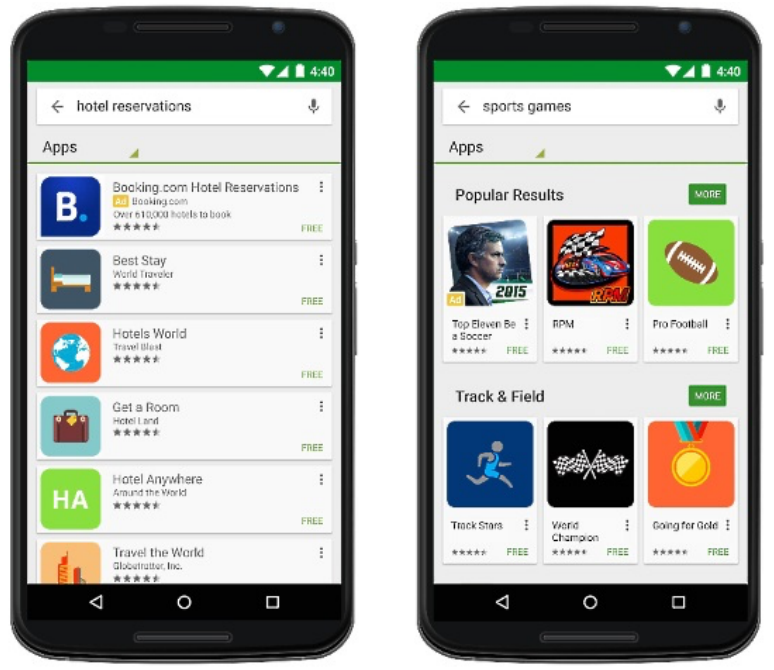
Real-world RL is hard.



Many RL successes in controlled domains.

How can RL add value in the real world?

ChatGPT



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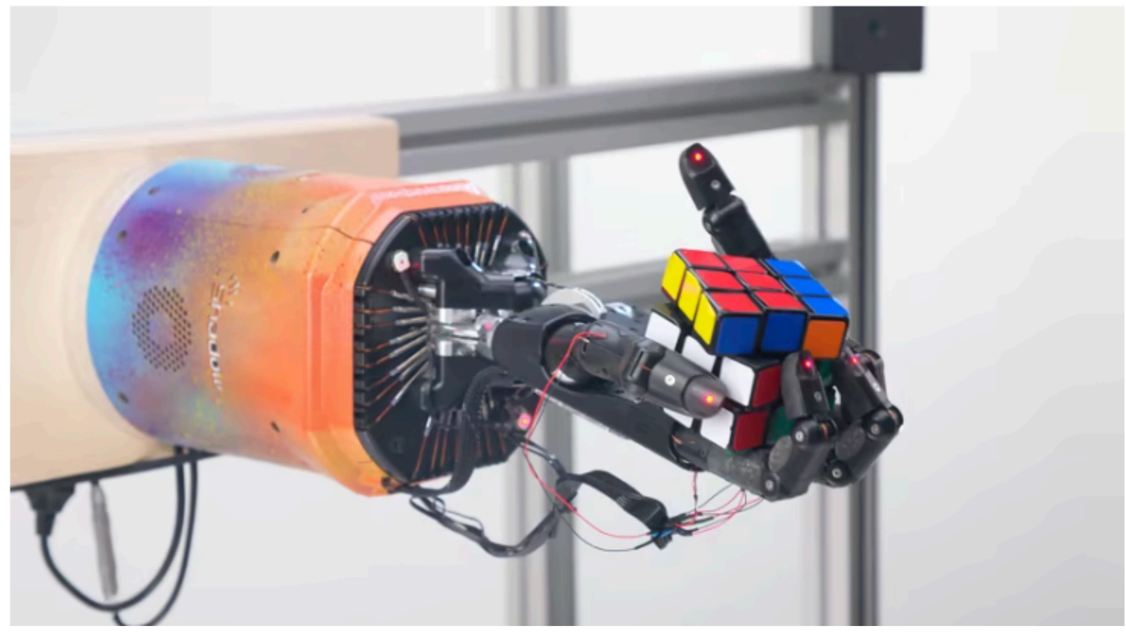

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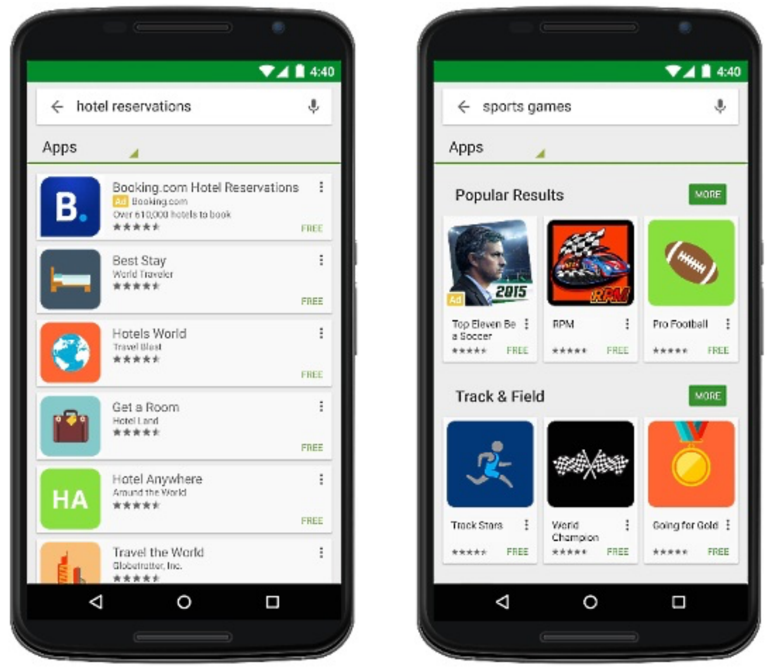

Issues:
sample complexity?
how to use offline data?
exploration/counterfactual reasoning?

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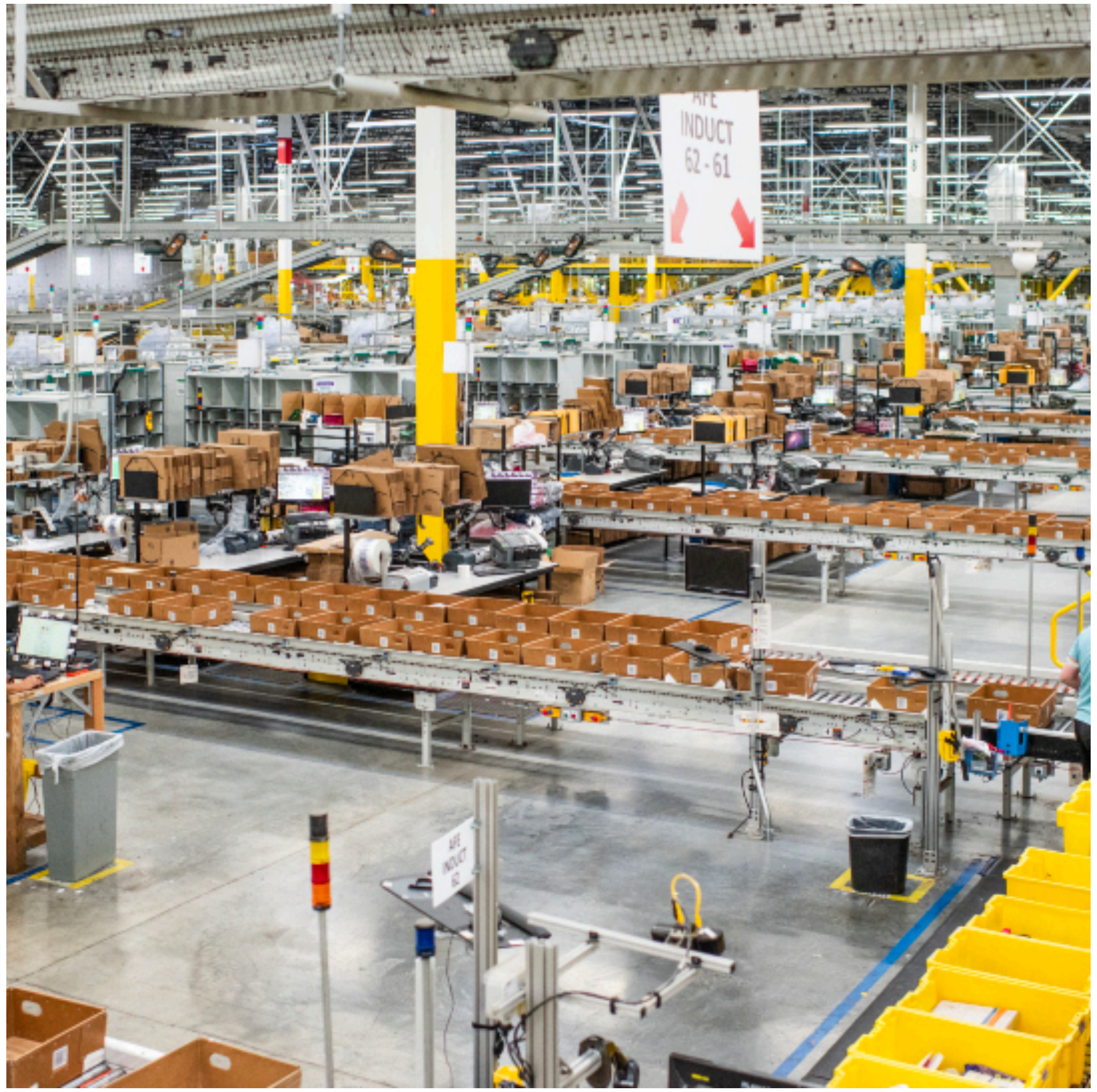


The Supply Chain Problem

- Supply Chain is about buying, storing, and transporting goods.
- There is a lot of historical “off-policy” data
    - e.g. Amazon, …
- Today: how can we use this data to solve the inventory management problem?
    - counterfactual issues?

Outline

Can we use historical data to solve inventory management problems in supply chain?

- How to use historical data?
- Moving to real-world inventory management problems
- Real world results

Largely based on this paper:
arxiv/2210.03137

Deep Inventory Management

Dhruv Madeka
Amazon, maded@amazon.com

Kari Torkkola
Amazon, karito@amazon.com

Carson Eisenach
Amazon, ceisen@amazon.com

Anna Luo
Pinterest*, annaluo676@gmail.com

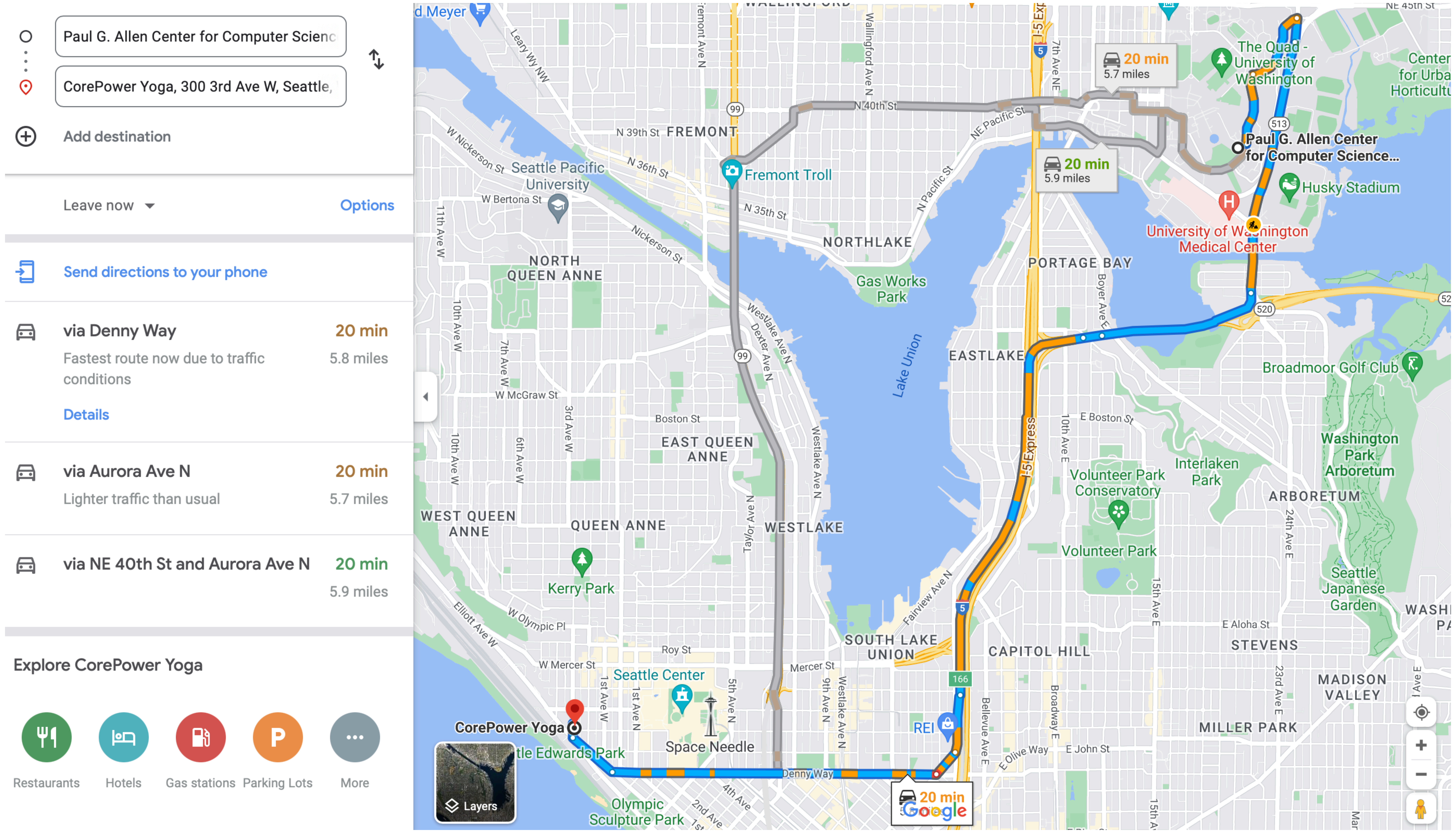
Dean P. Foster
Amazon, foster@amazon.com

Sham M. Kakade
Amazon, Harvard University, shamisme@amazon.com

I: Utilizing historical data

Warm up: Vehicle Routing

(when using historical data might be ok)



Warm up: Vehicle Routing

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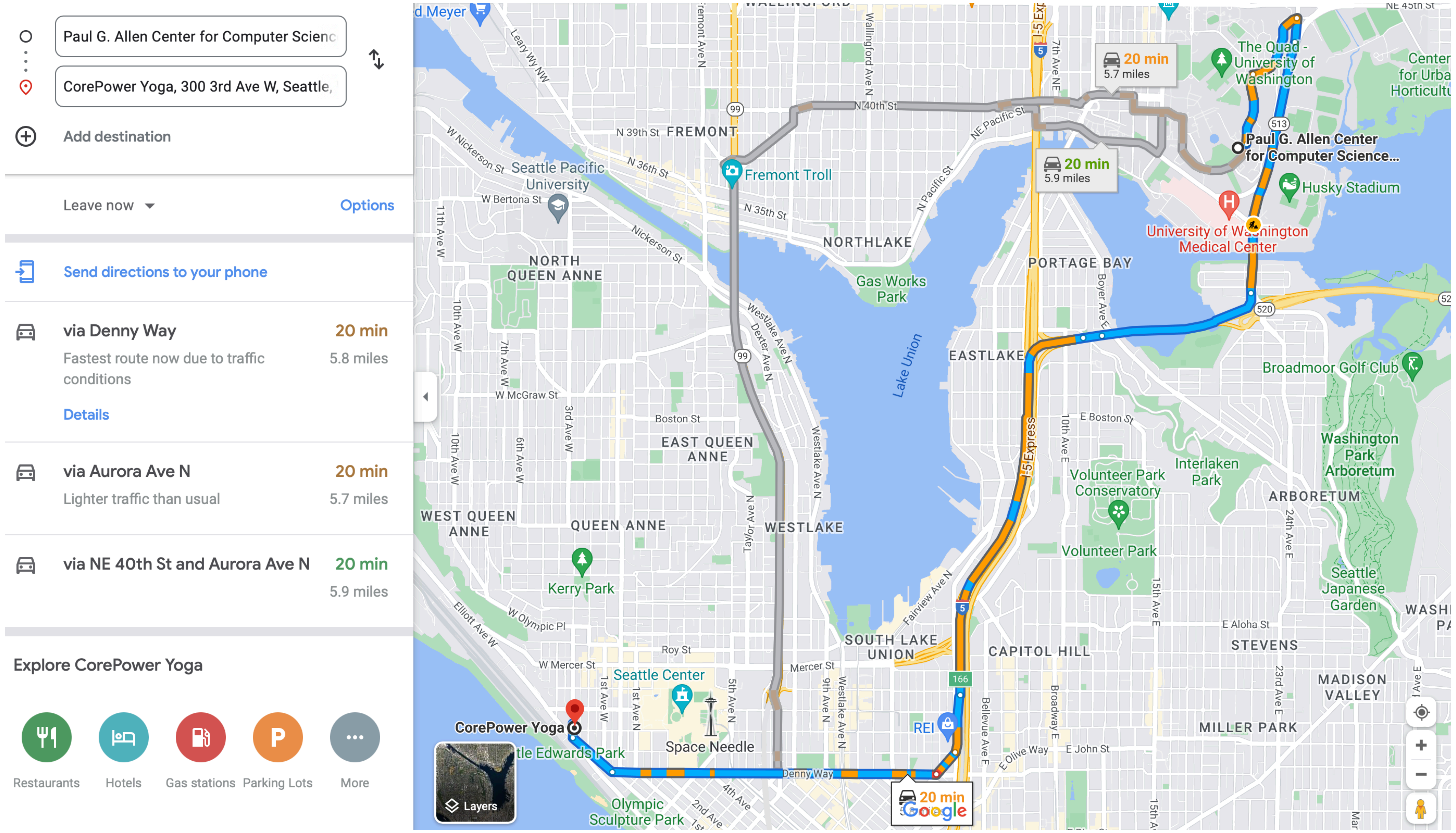
- We want a good policy for routing a single car.



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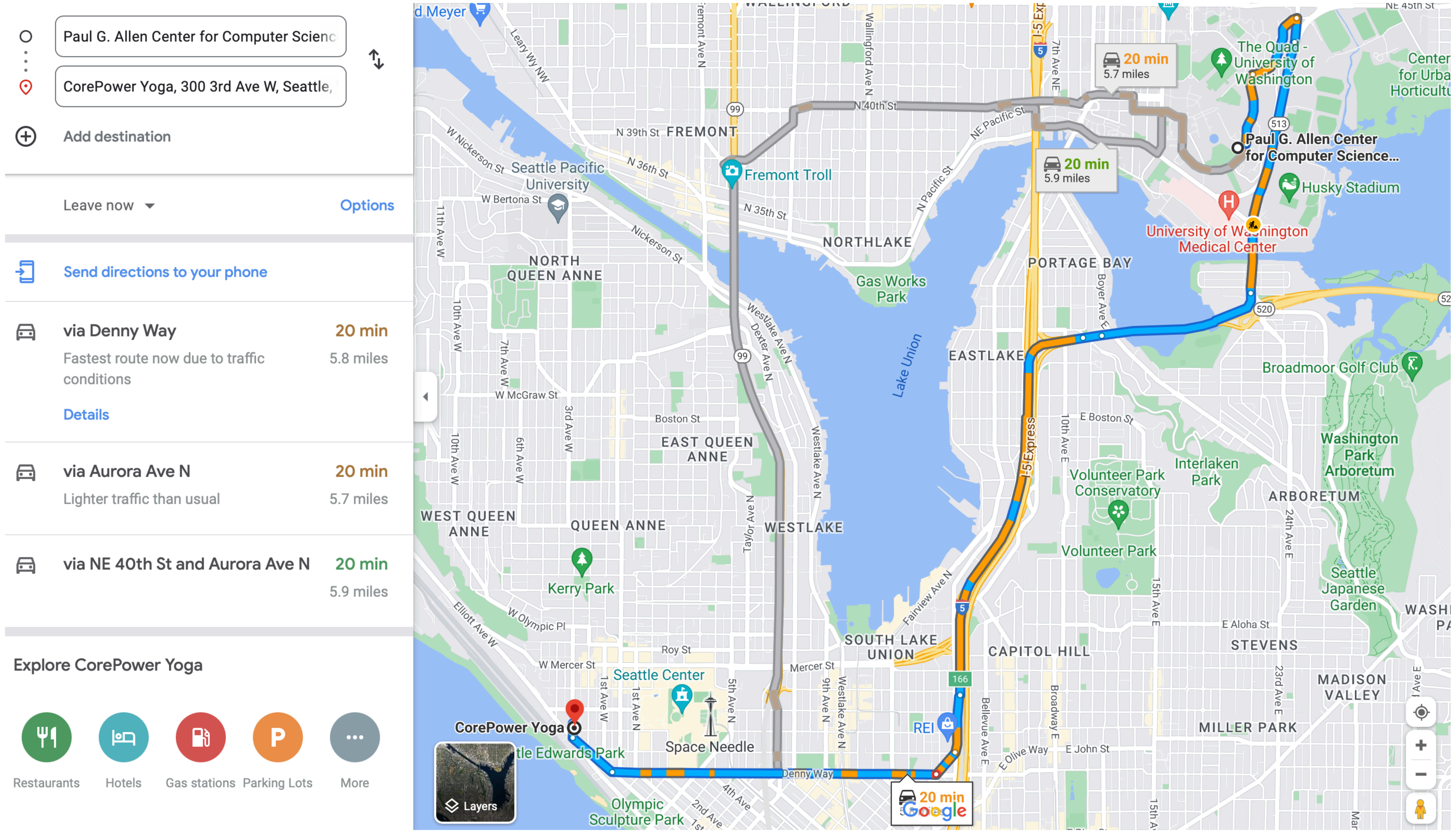
  features: time of day, holiday indicators, current traffic, sports games, accidents, location, weather,



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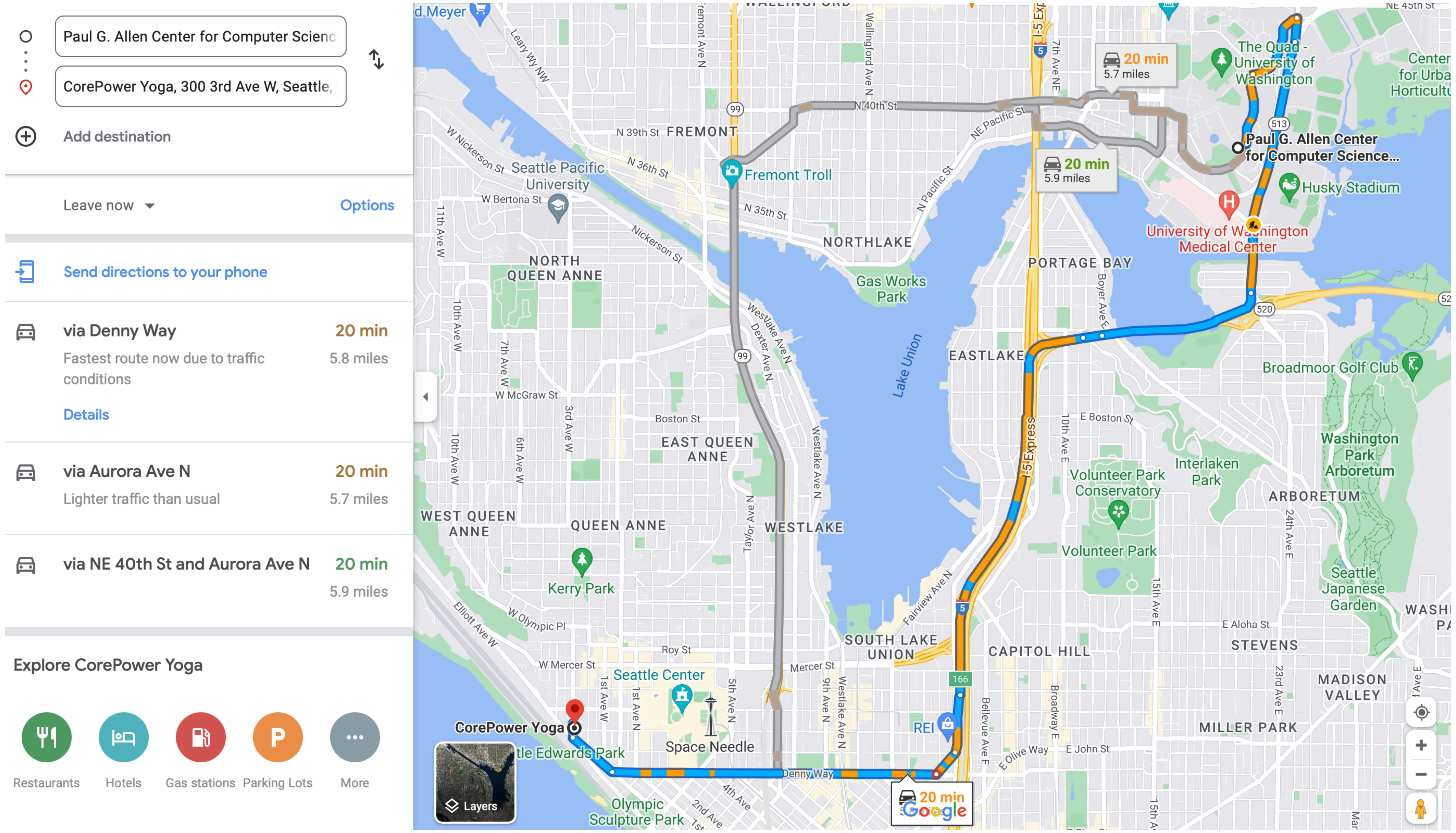

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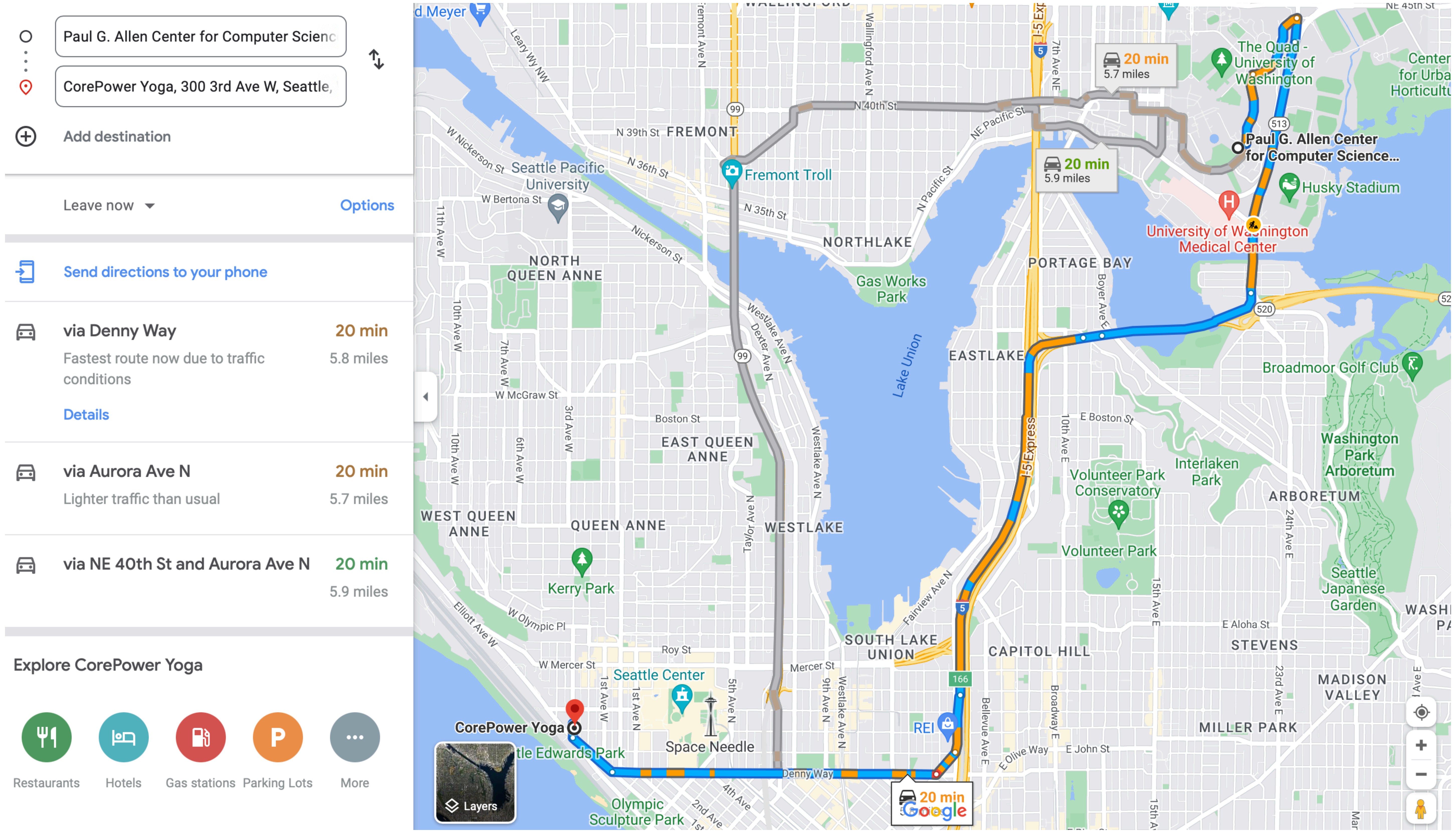

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- Backtesting policies:
  - Key idea: a single route minimally affects traffic
  - Counterfactual: with the historical data, we can see what would have happened with another policy.

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Cost= $1

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2	120	60	-	120

Supply Chain Data

Price= $2
Cost= $1

Time	Inventory	Demand	Order	Revenue
0	100	20	-	40
0	80	-	10	-10
1	90	20	-	40
1	70	-	50	-50
2	120	60	-	120
2	60	-	10	-10

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80 + 40

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- Current order doesn't impact future demand.
  - This allows us to backtest!
  - Empirically, backlog due to unmet demand does not look significant.[1]

1. See Verhoef et al (2006)

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- Exogenous MDPs: Growing literature around a class of MDPs where a large part of the state is driven by an exogenous noise process [Efroni et al 2021, Sinclair et al 2022]

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  - Known controllable part (inventory) I_t: (known) evolution is dependent on our action.
    - $I_t = \max(I_{t-1} + a_{t-1} - D_t, 0)$ (and suppose we start at I_0).
  - Immediate reward is the profits: $r(s_t, I_t, a_t) :=$ Price$_t \times \min($Demand$_t, I_t) -$ Cost$_t \times a_t$

"rl state" : (I_t, s_t)

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$$V_H(\pi) = E_\pi\left[\sum_{t=1}^{H} \gamma^t r(s_t, I_t, a_t)\right]$$

Why is it an interesting RL problem?

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- Lots of time dependence!
  - If you buy too much, you're left with the inventory for months!
  - Your actions (orders) affect the state at a random time later
  - Tons of correlation across time (Demand, Price, Cost, Seasonality, etc)

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Theorem: RL in ExoMDPs is as easy as Supervised Learning

Suppose we have K policies $\Pi = \{\pi_1, \ldots \pi_K\}$, and we have N sampled exogenous paths. Then we can accurately backtest up to nearly $K \approx 2^N$ policies.

Formally, for $\delta \in (0,1)$, with pr. greater than $1 - \delta$ - we have that for all $\pi \in \Pi$:

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(assuming the reward r_t is bounded by 1).

- Implications:
  - We can optimize a neural policy on the past data.
  - In the usual RL setting (not exogenous), we would have an amplification factor of (at least) $\min\{2^H, K\}$, using historical data due to the counterfactual issue.

II: Real World Inventory Management Problems

Real-world Issue: Censored Demand

- When demand $\geq$ inventory, what customers see:

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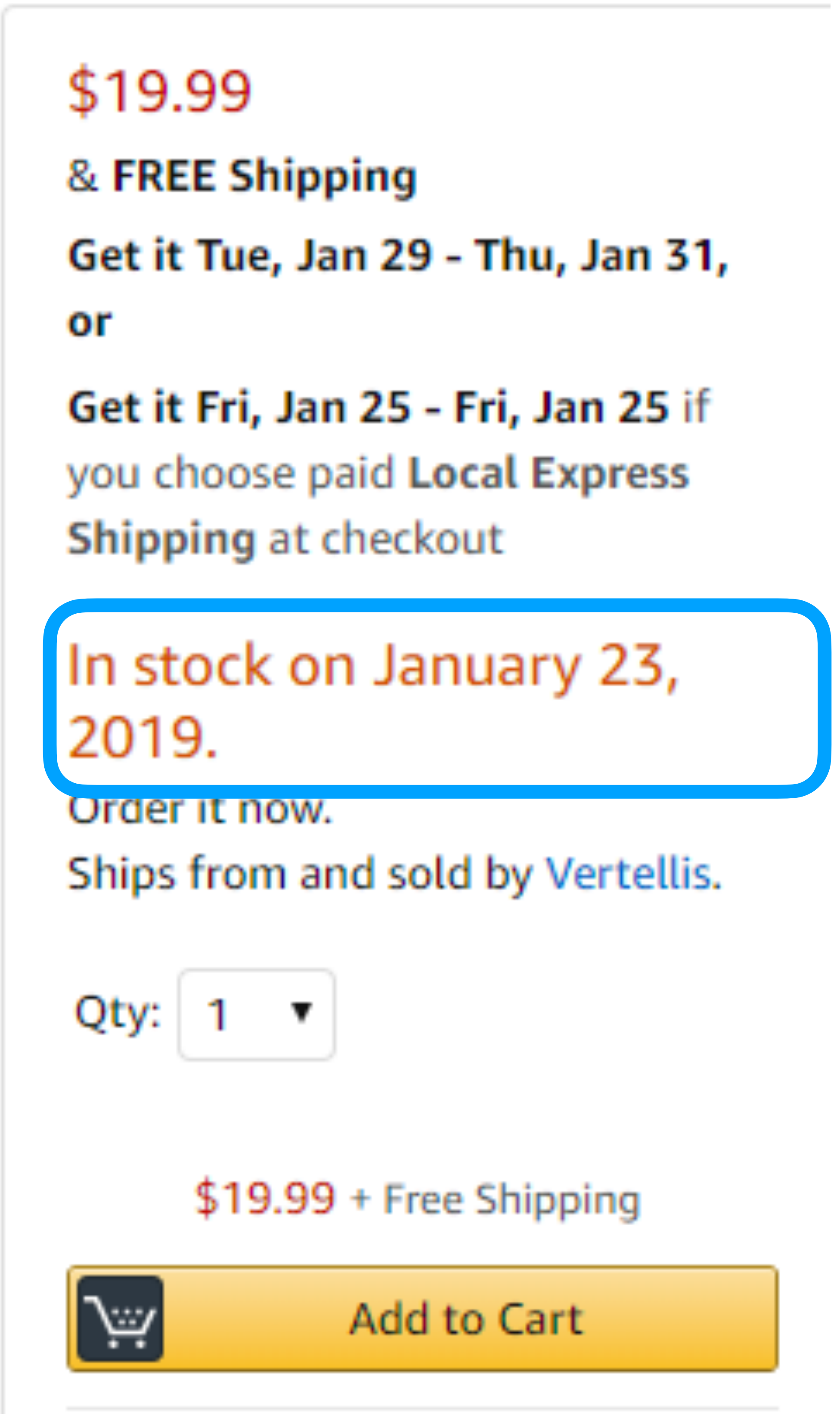

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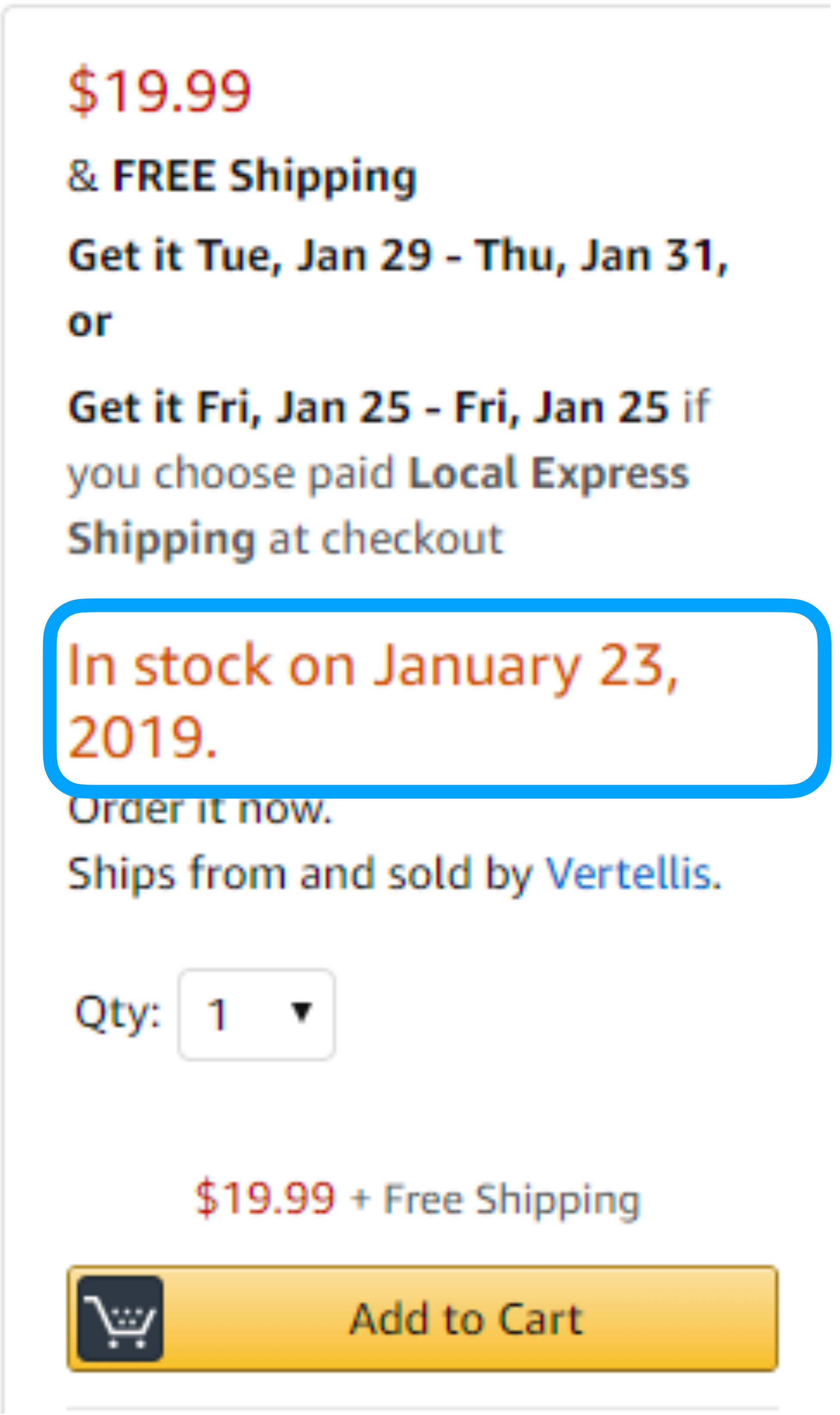

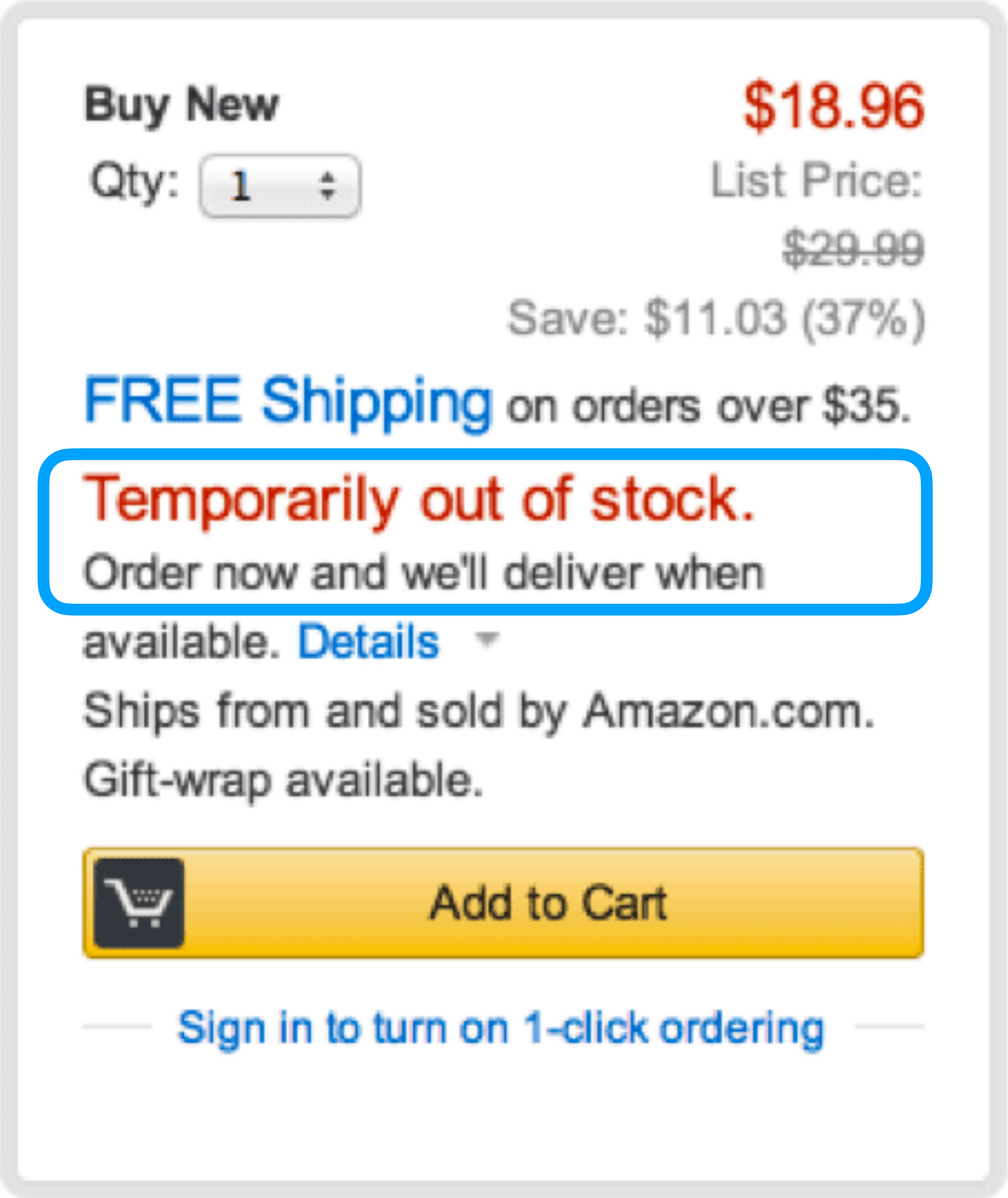

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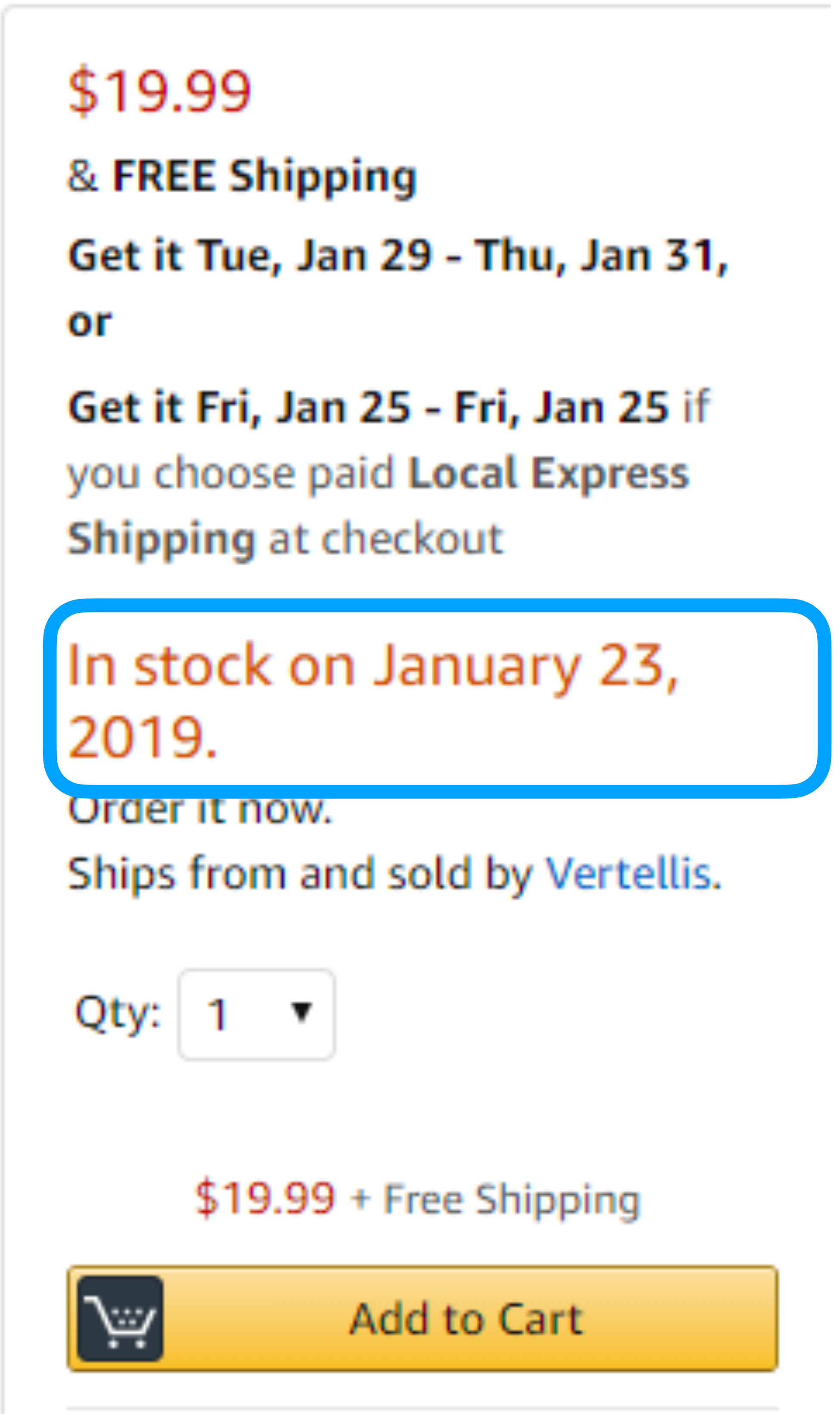

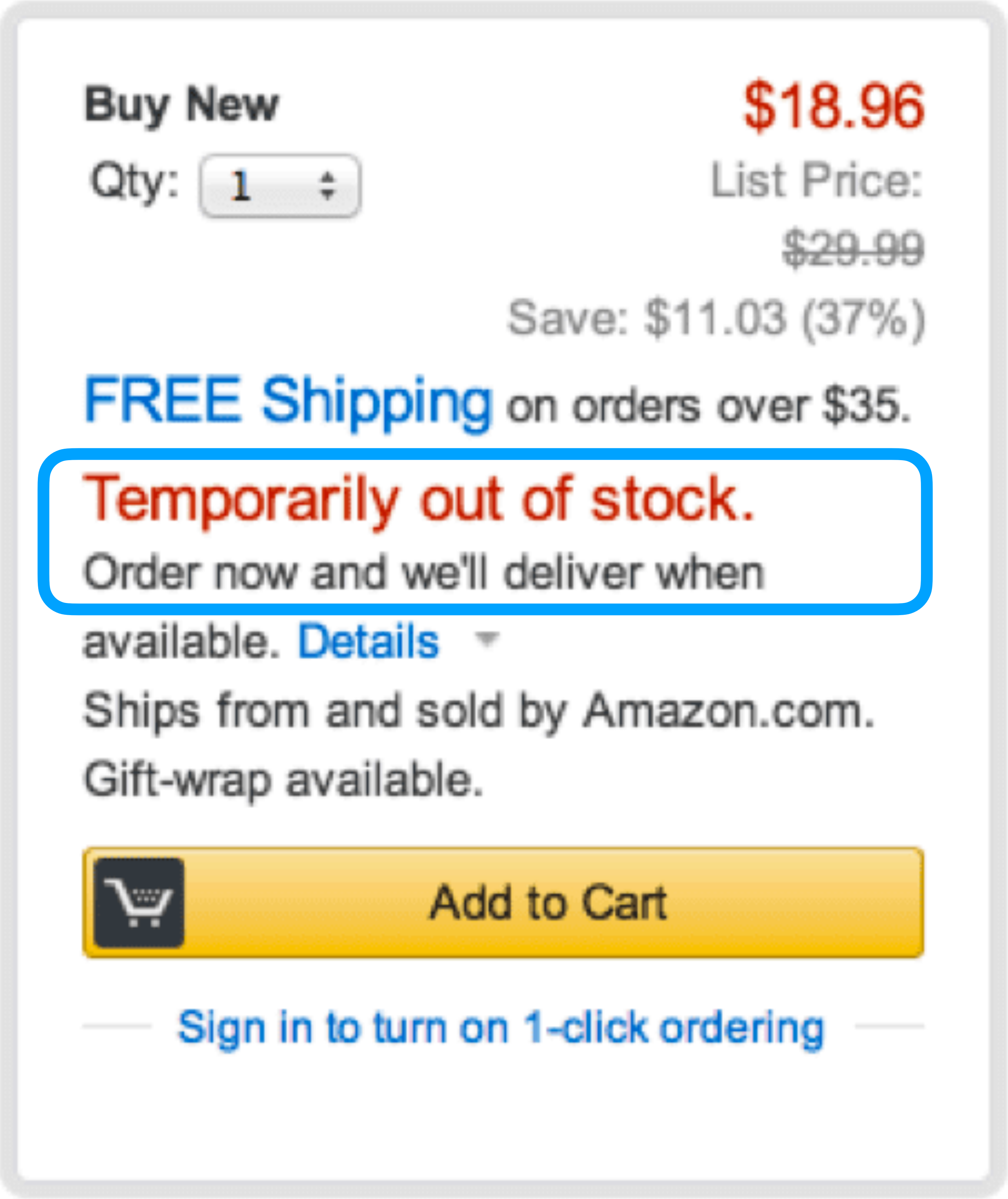

We only observe **sales** not the **demand:**

Sales $:=$ min(Demand, Inventory)

Can we still backtest?

Our historical data is then censored….

$$\text{Sales} := \min(\text{Demand}, \text{Inventory})$$

Price= $2

Cost= $1

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Time	Inventory	True Demand	Sales	Order	Revenue
T	10	??	10	-	20
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮

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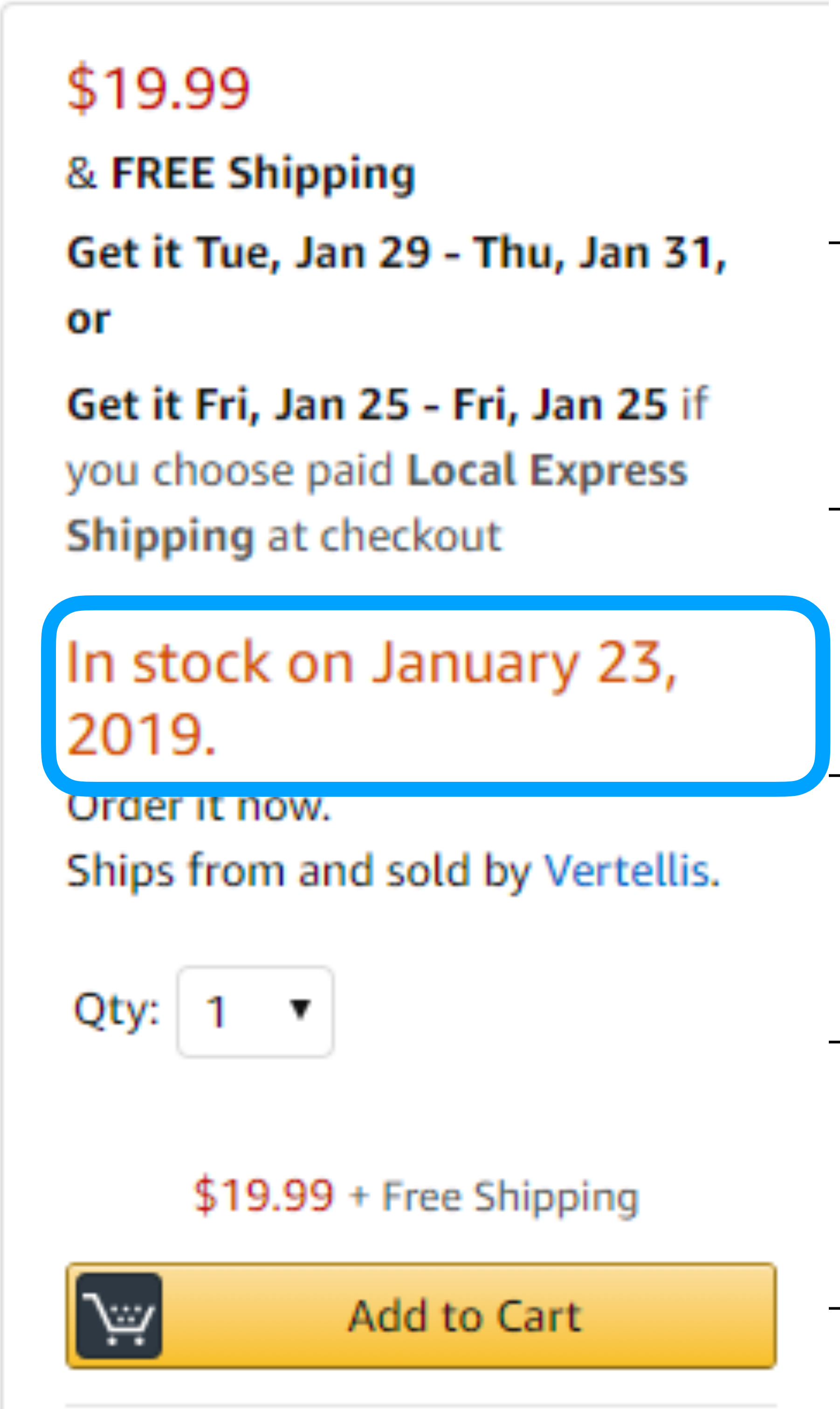

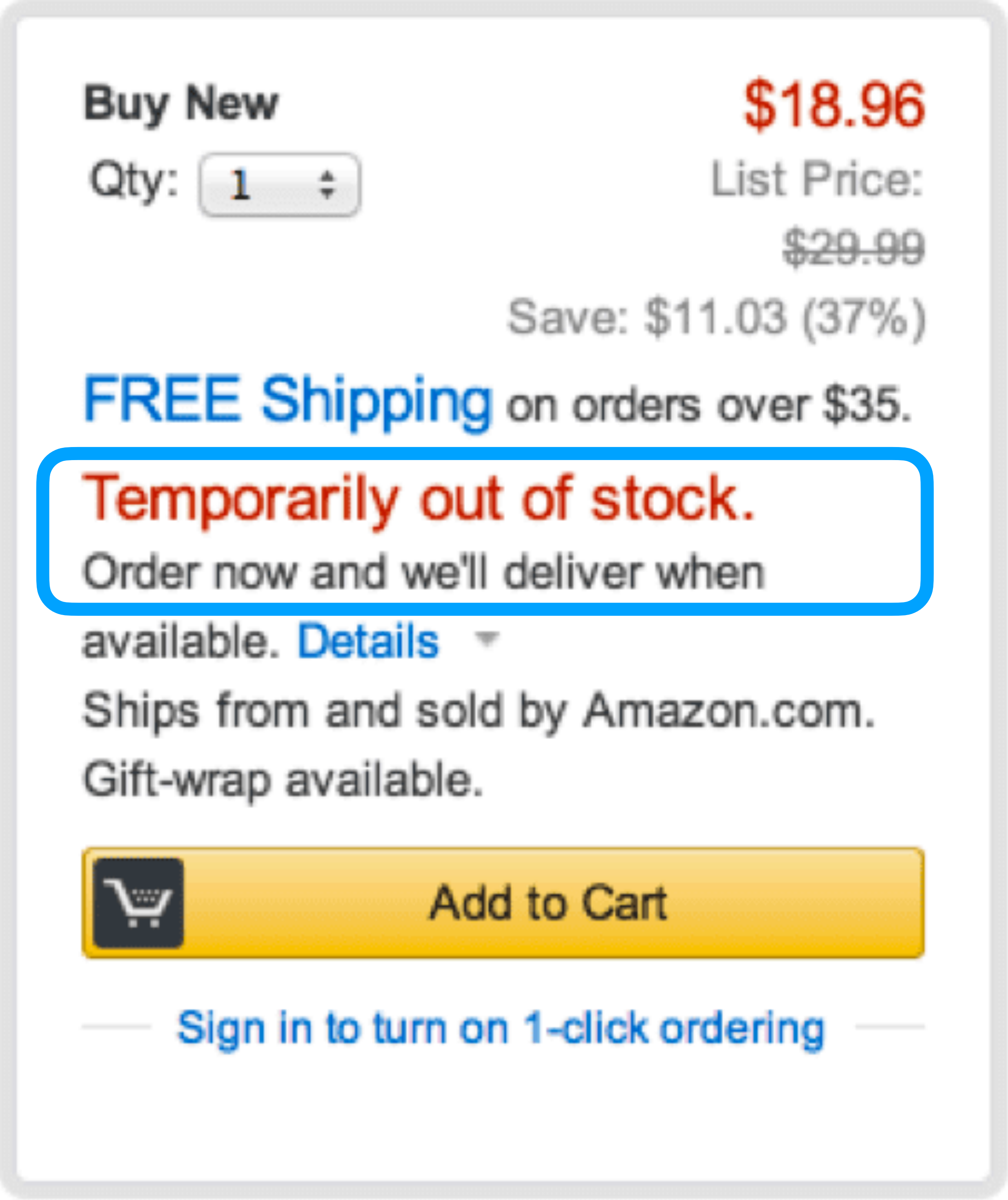

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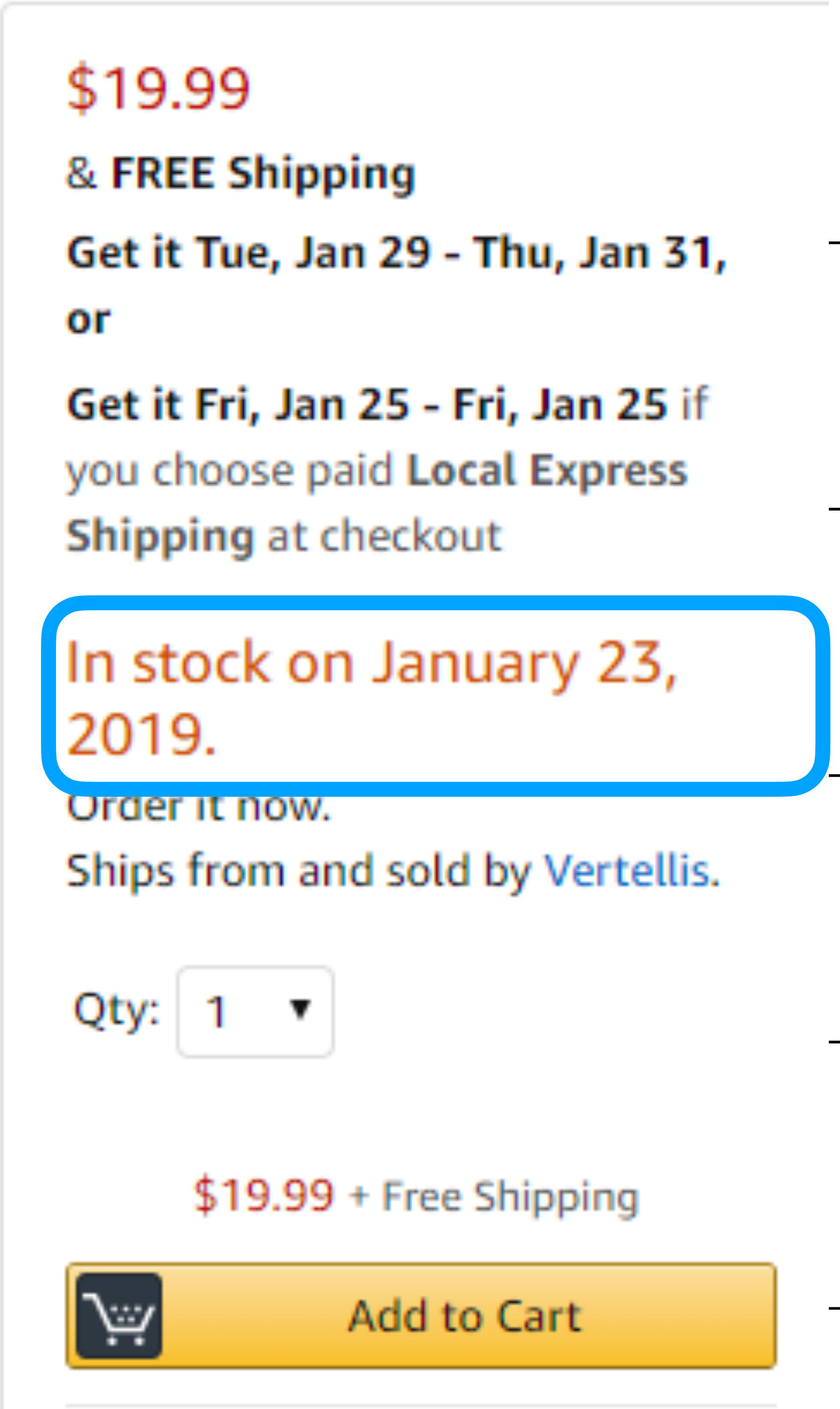
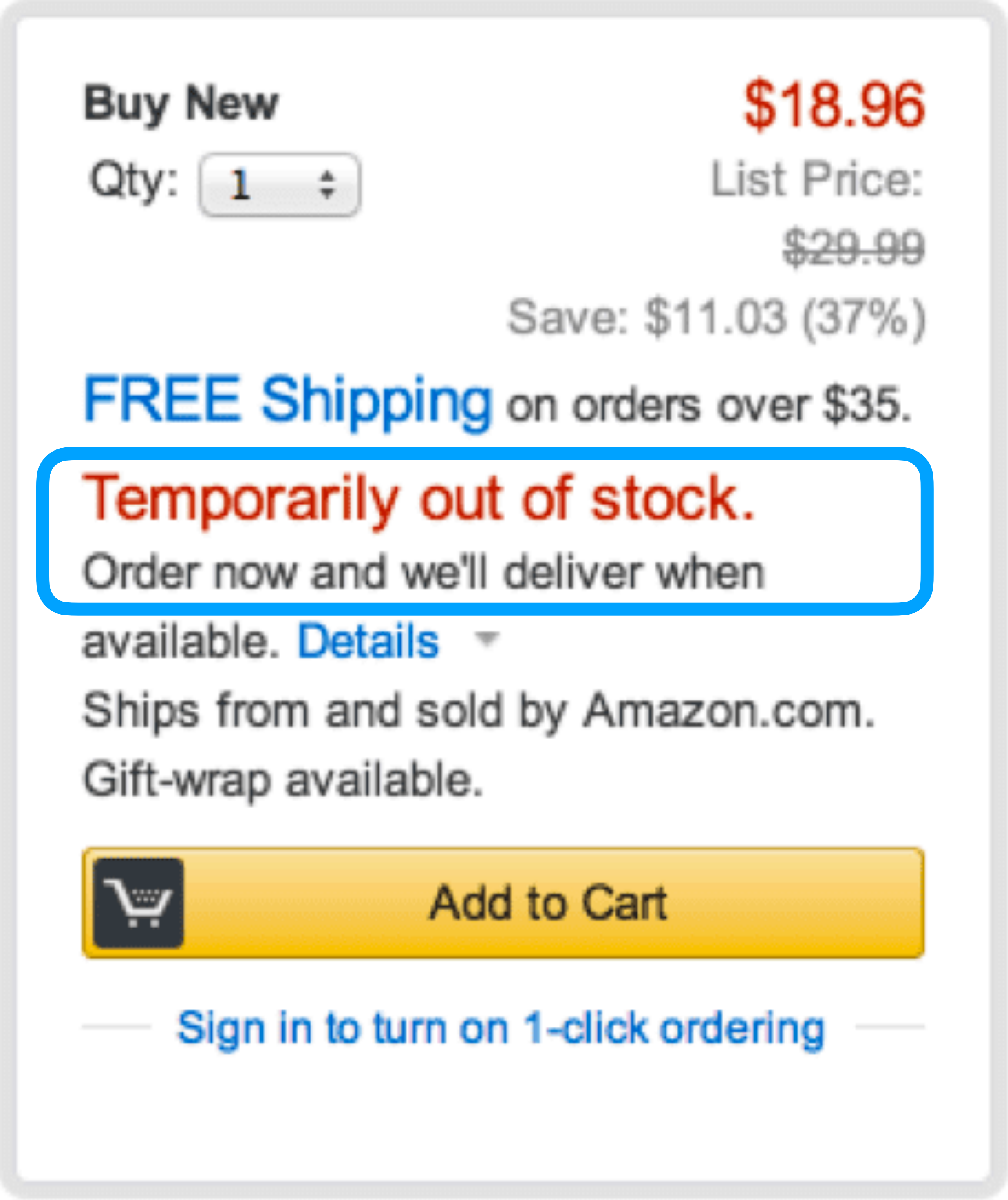
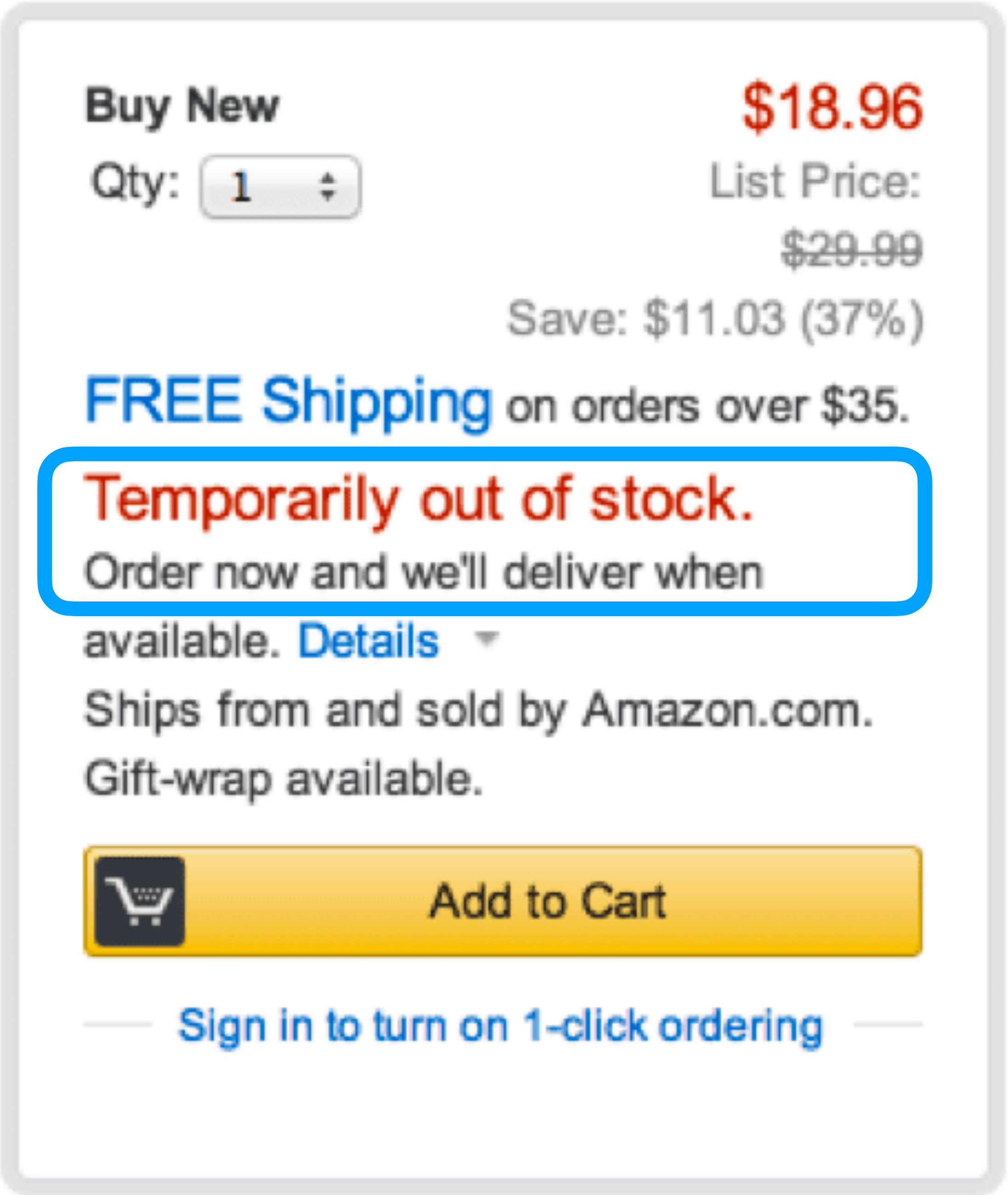
If we could fill in the missing demand, then we could still backtest!





We have many observed historical covariates

- Covariates:
  Sales, Web Site, Glance Views, Product Text, Reviews
- Example: the #times customers look at an item gives us info about the unobserved demand.



- Let's forecast the missing variables from the observed covariates!
  $\hat{\mathbb{P}}(\text{Missing Data} \mid \text{Observed Data})$

Uncensoring the data….

Sales := min(Demand, Inventory)

Price= $2
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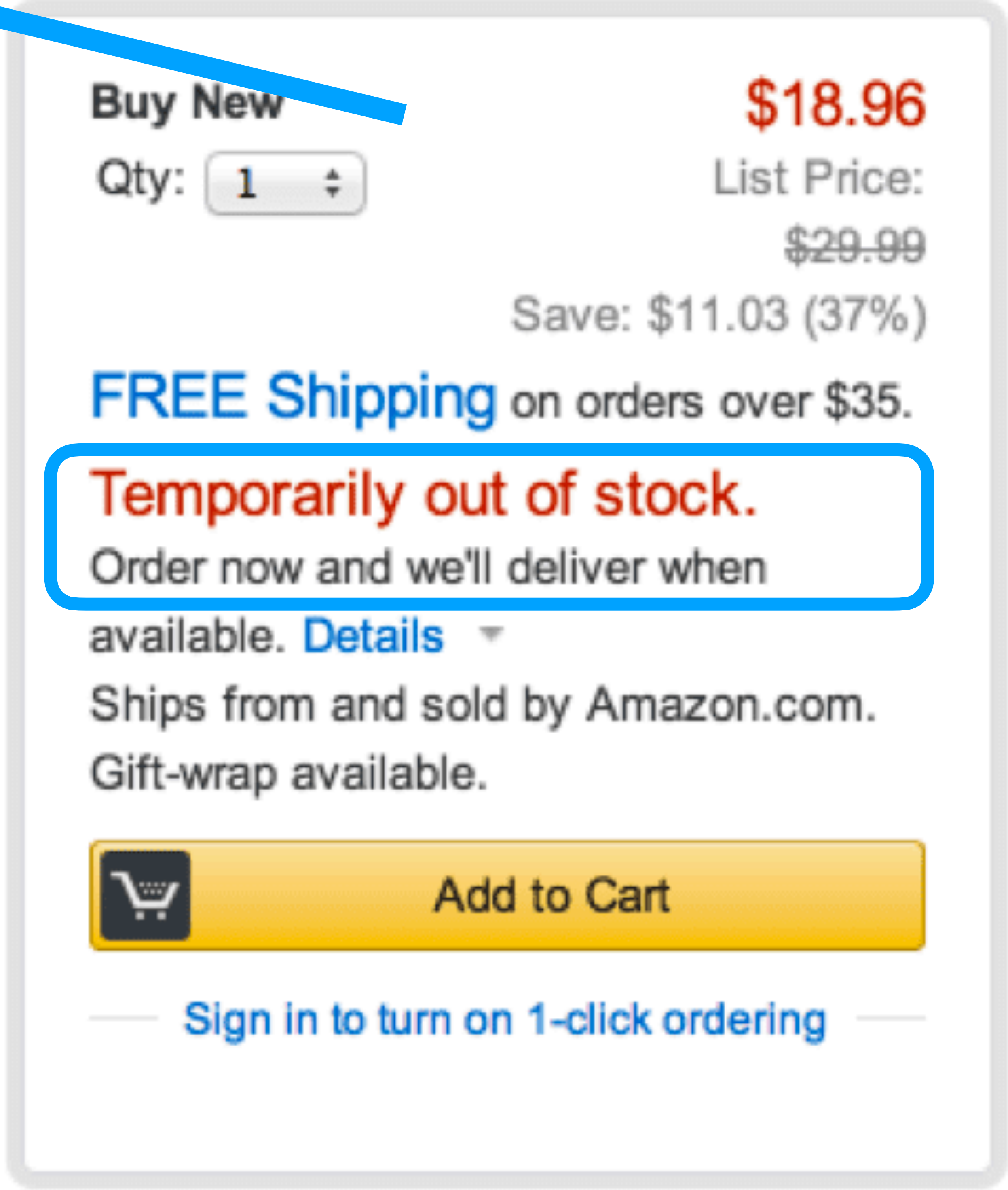

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⋮	⋮	⋮	⋮		
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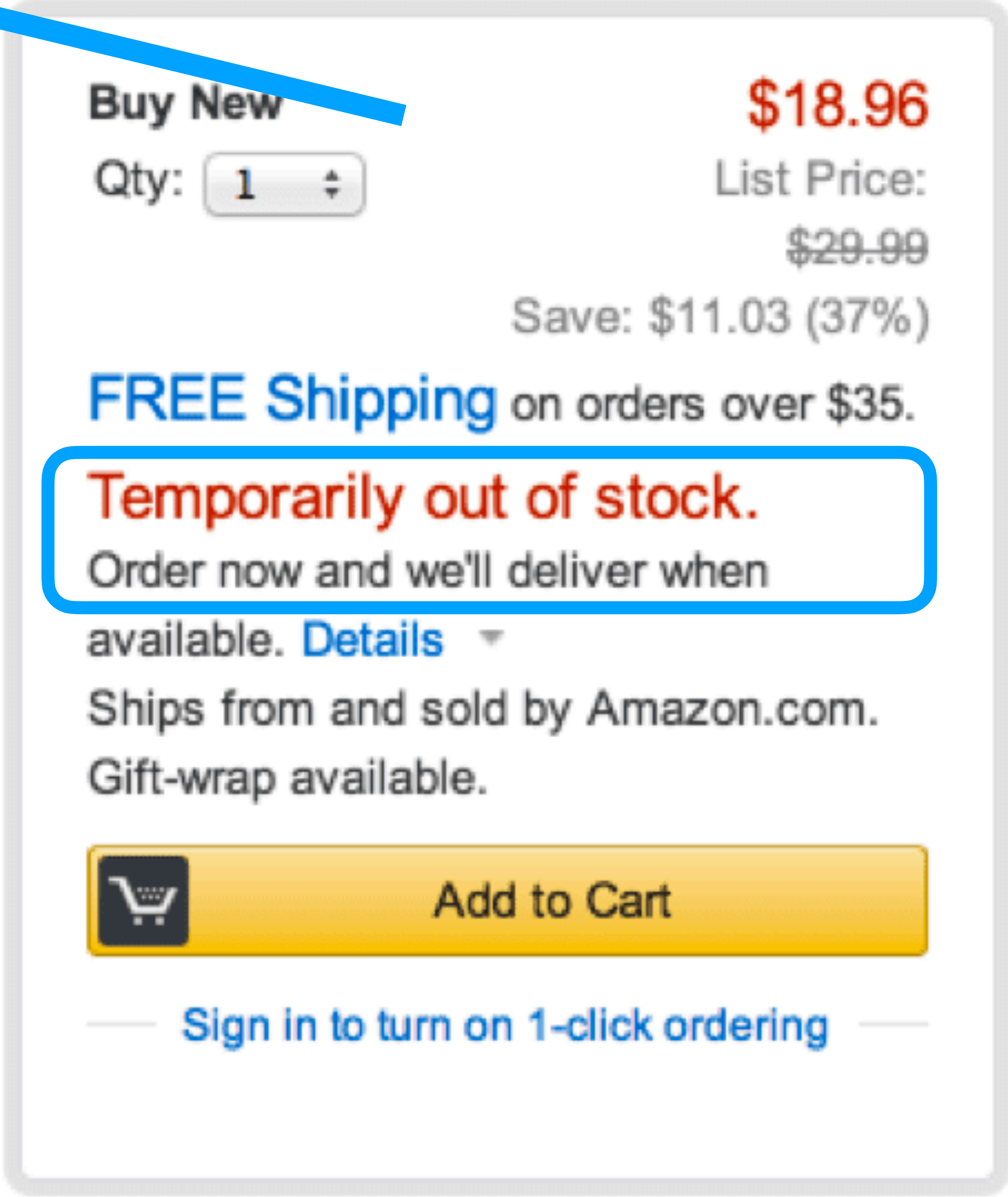

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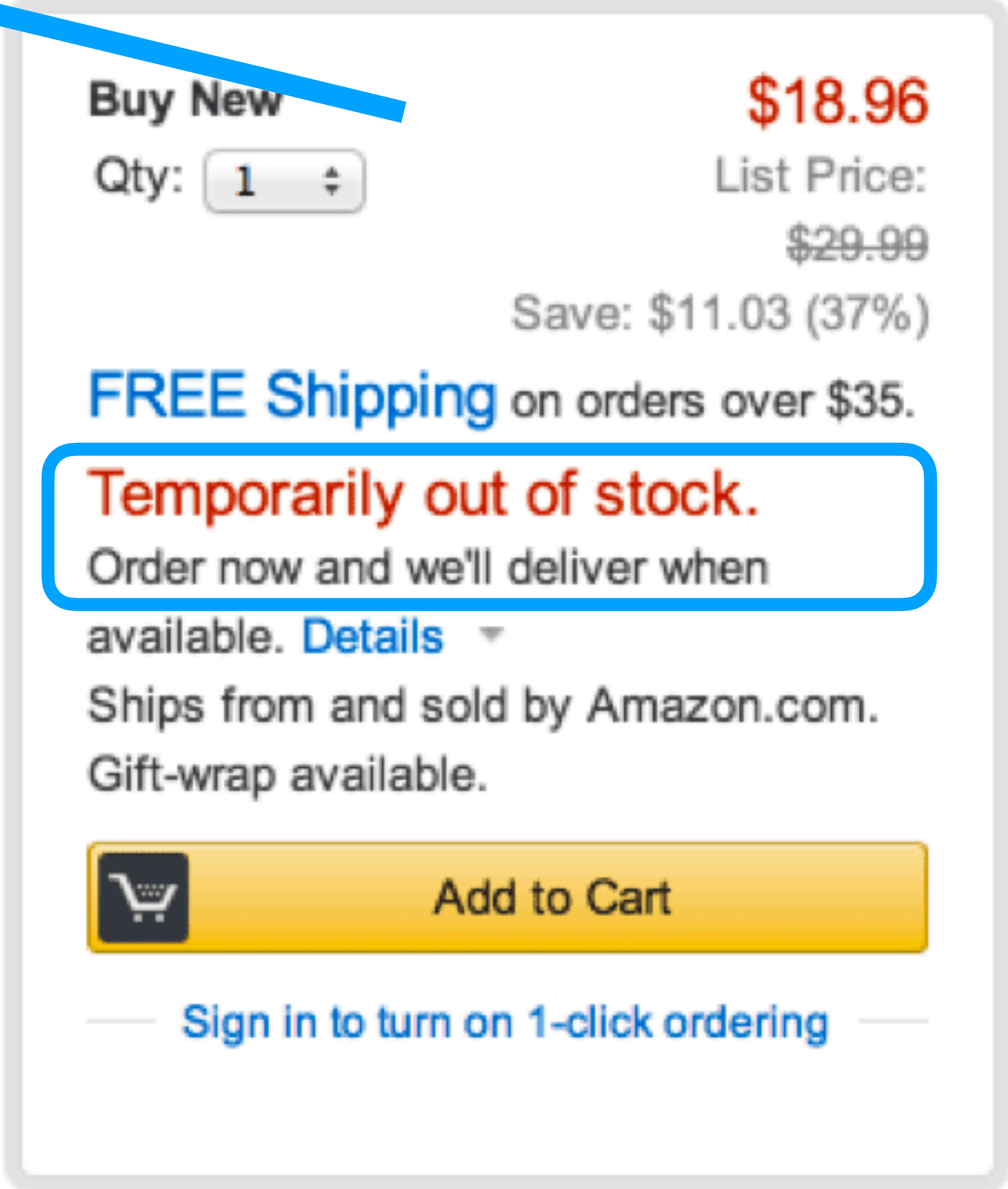

Key idea: Use covariates (e.g. glance views) to forecast missing demand, vendor lead times, etc

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Setting: we have N sampled sequences $\{s_1^i, s_2^i, \ldots s_H^i\}_{i=1}^N$,
where M_i and O_i are the missing and observed exogenous variables in sequence i.

Forecast: $\widehat{\mathbb{P}}^i = \widehat{\Pr}(M_i \mid O_i)$ is our forecast of $\mathbb{P}^i = \Pr(M_i \mid O_i)$.

Assume: With pr. 1, forecasting has low error: $\frac{1}{N}\sum_{i=1}^{N} \text{TotalVar}\left(\mathbb{P}^i, \widehat{\mathbb{P}}^i\right) \le \epsilon_{\text{sup}}$.

Guarantee: For any $\delta \in (0,1)$, with pr. greater than $1-\delta$, for all $\pi \in \Pi$:

$$|V_0(\pi) - \hat{V}_0(\pi)| \le H\left(\epsilon_{\text{sup}} + \sqrt{\frac{\log(K/\delta)}{N}}\right)$$

III: Training Policies & Empirical Results

The Simulator

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- Collection of historical trajectories:
  - 1 million products
  - 104 weeks of data per product

Data

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← Data

← Corrections

The Simulator

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  - Demand
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- Policy gradient methods in a "gym":
  - "gym" ↔ backtesting ↔ simulator
    (note the "simulator" isn't a good world model).
  - The policy can depend on many features.
    (seasonality, holiday indicators, demand history, product details, text features)

Data

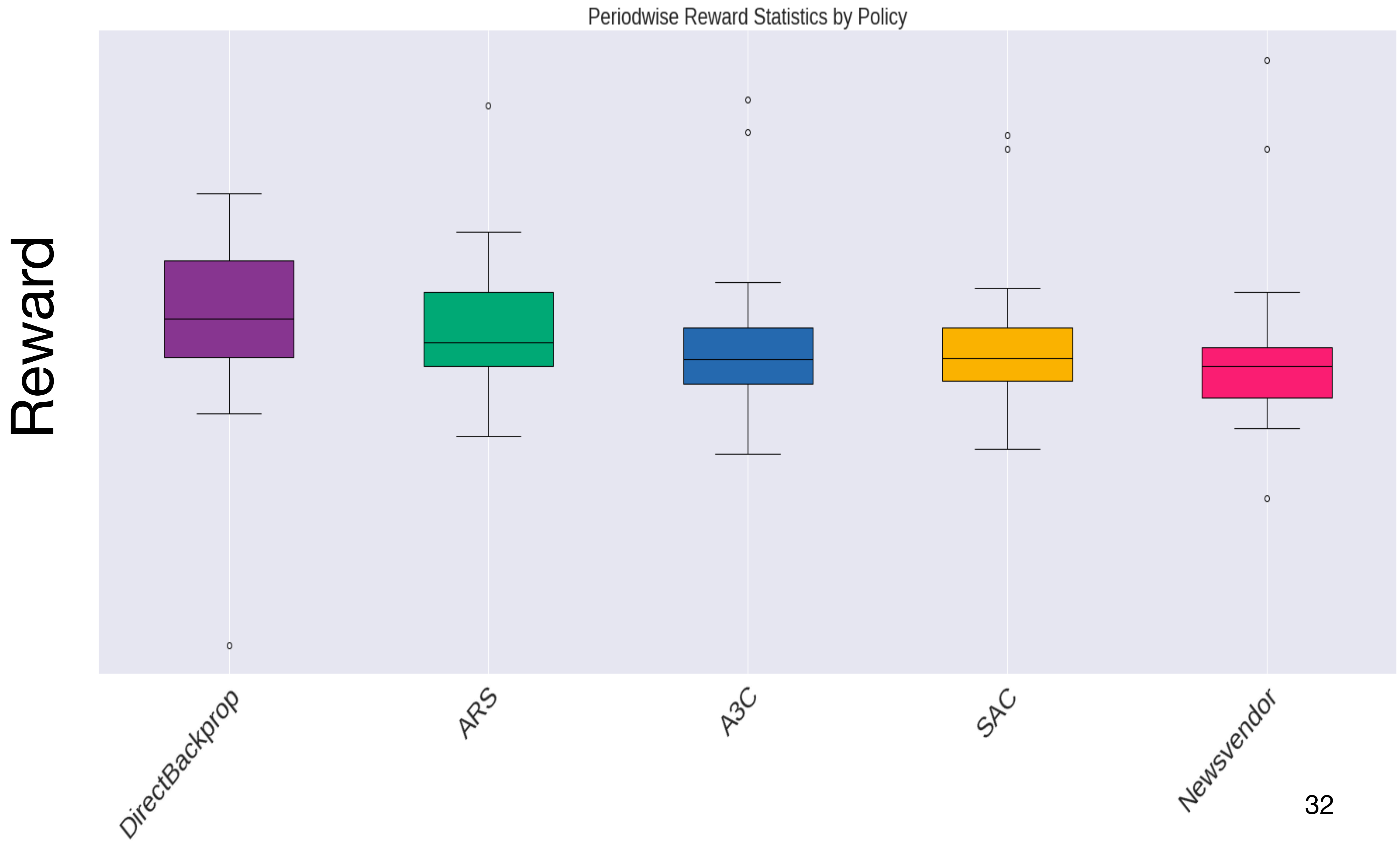
Corrections

Simulator

Sim to Real Transfer

- Sim: the backtest of DirectBackprop improves on Newsvendor.
- Real: DirectBackprop significantly reduces inventory without significantly reducing total revenue.

Simulation

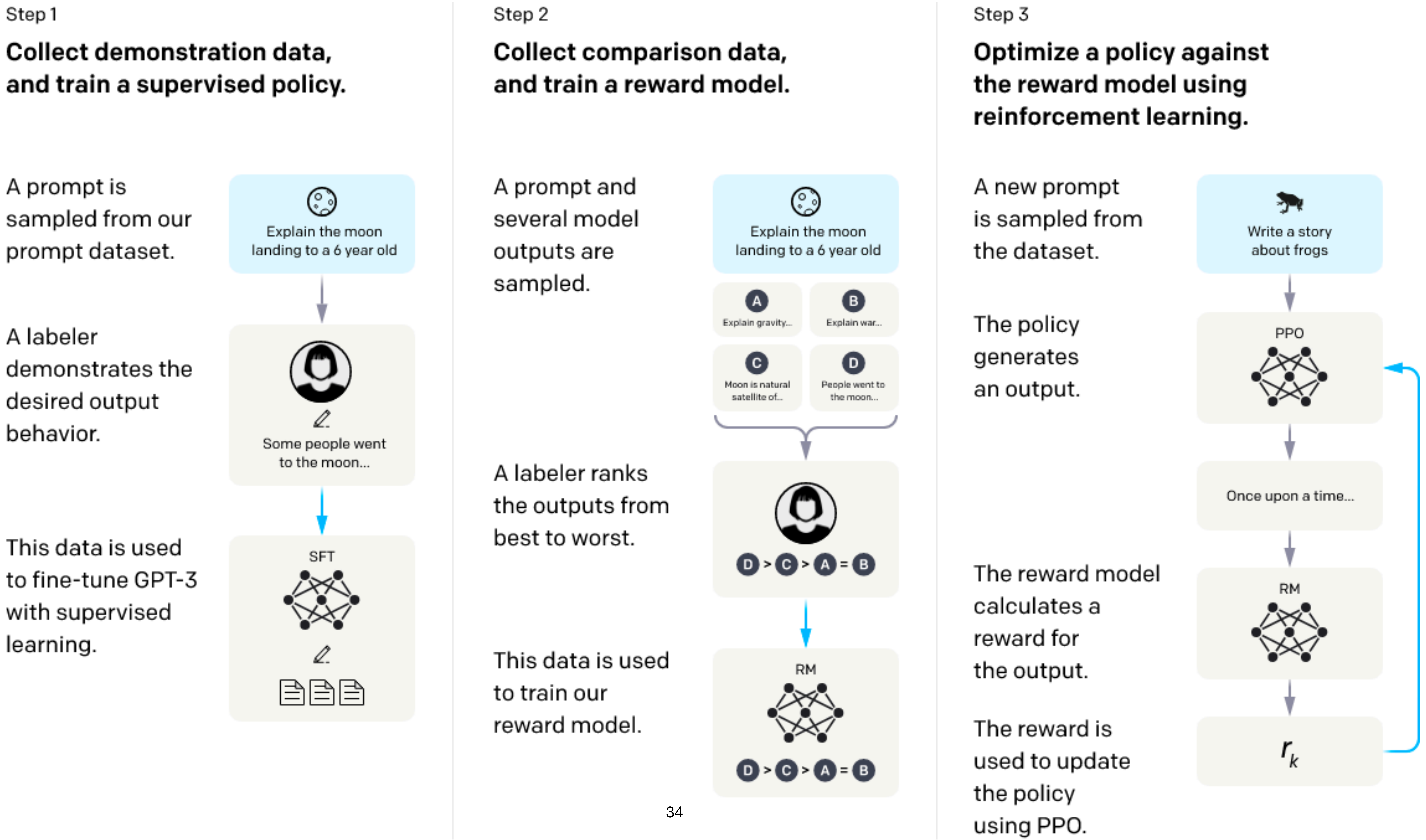


Real World

Metrics	% change
Inventory Level	**-12** $\pm$ 6
Revenue	**2.6%**

RLHF

RL from Human Feedback (RLHF)



Summary:

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bit.ly/3RcTC9T

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- The Course: sequential decision making (causality + decisions)
    - RL gives a helpful set of tools.
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Summary:

Today: adding context to bandits requires SL but makes it much more useful

- The Course: sequential decision making (causality + decisions)
  - RL gives a helpful set of tools.
  - RL also gives an interesting viewpoint.
- **We hope you enjoyed the course!**

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Both cases allow a version of linUCB by extension of the same ideas: fit coefficients via least squares and use Chebyshev-like uncertainty quantification to get UCB

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For $t = 0 \rightarrow T - 1$

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2. Observe x_t & choose $a_t = \arg\max_k \left\{ \phi(x_t, k)^\top \hat{\theta}_t + c_t \sqrt{\phi(x_t, k)^\top A_t^{-1} \phi(x_t, k)} \right\}$

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For $t = 0 \to T-1$

1. $\forall\, k$, define $A_t = \sum_{\tau=0}^{t-1} \phi(x_\tau, a_\tau)\phi(x_\tau, a_\tau)^\top + \lambda I$ and $\hat{\theta}_t = A_t^{-1} \sum_{\tau=0}^{t-1} \phi(x_\tau, a_\tau) r_\tau$

2. Observe x_t & choose $a_t = \arg\max_k \left\{ \phi(x_t, k)^\top \hat{\theta}_t + c_t \sqrt{\phi(x_t, k)^\top A_t^{-1} \phi(x_t, k)} \right\}$

3. Observe reward $r_t \sim \nu^{(a_t)}(x_t)$

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iii. The other formulation, with separate $A_t^{(k)}$ and $\hat{\theta}_t^{(k)}$, is called disjointed

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But in principle, there is no “free lunch”, i.e., the hardness of the problem now transfers over to choosing a good model (a bad model will lead to bad performance)