Reinforcement Learning & Markov Decision Processes

Lucas Janson and Sham Kakade

CS/Stat 184: Introduction to Reinforcement Learning

Fall 202~~2~~ 3

Today

- Logistics (Welcome!)
- Overview of RL
- Markov Decision Processes
  - Problem statement
  - Policy Evaluation

Course staff introductions

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- Homework 0 is posted today!
  - This is “review” homework for material you should be familiar with to take the course.

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- Project (30%): 2-3 people per project. Will be empirical.

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- Regrading: ask us in writing on Ed within a week

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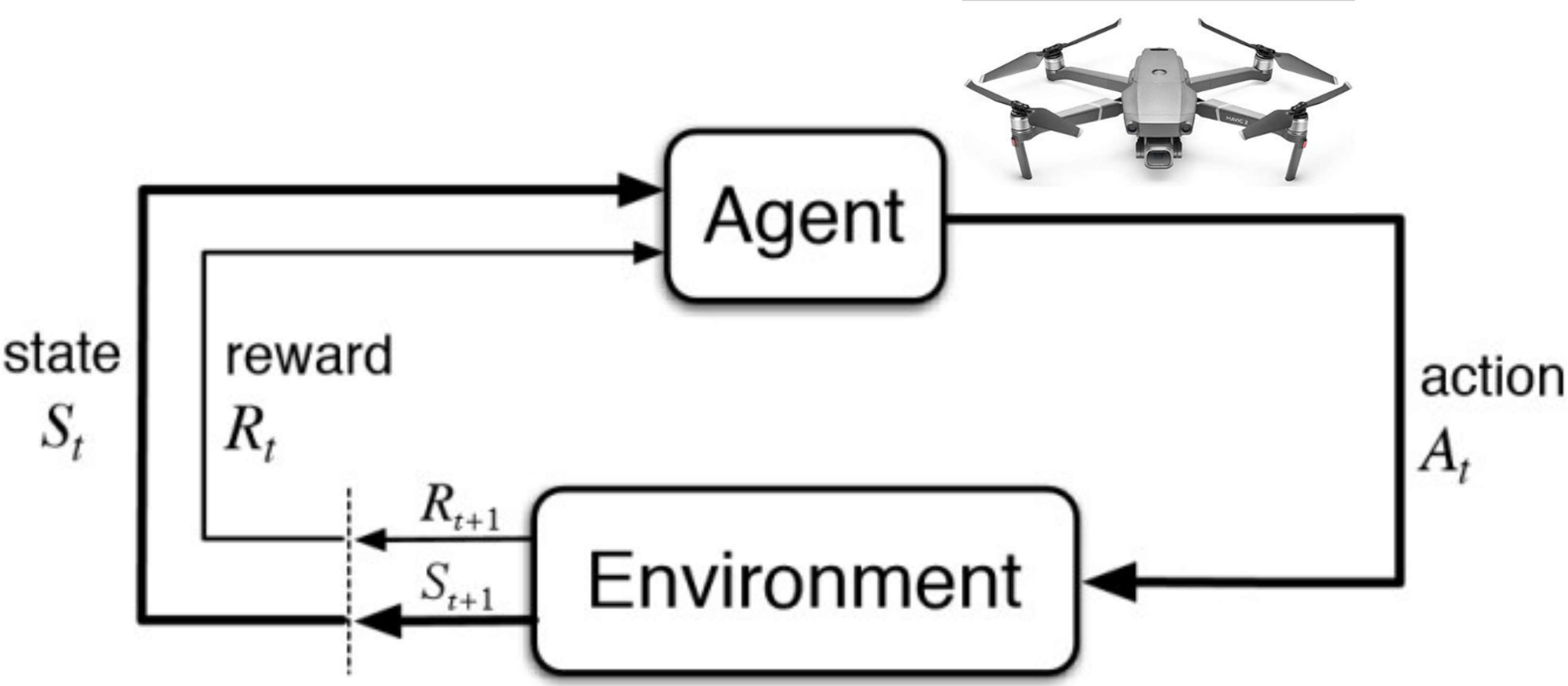
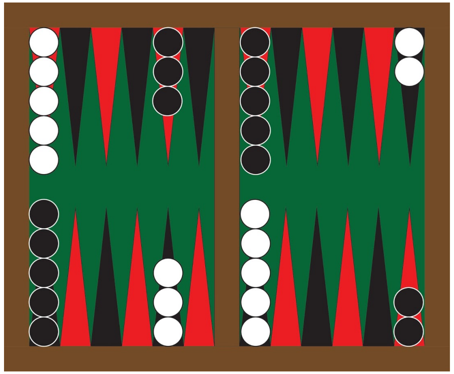
The RL Setting, basically



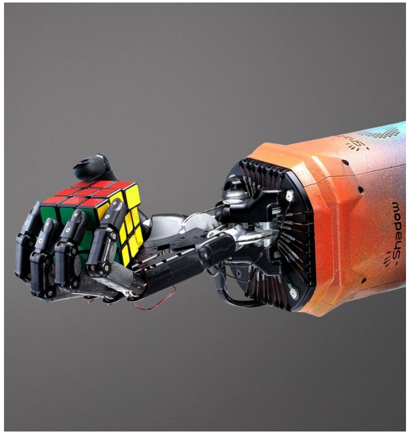
Many RL Successes



TD GAMMON [Tesauro 95]

[AlphaZero, Silver et.al, 17]

[OpenAI Five, 18]



[OpenAI,19]

Supply Chains [Madeka et al '23]

Many Future RL Challenges

Vs Other Settings

	Learn from Experience	Generalize	Interactive	Exploration	Credit assignment
Supervised Learning	✓	✓			
Bandits ("horizon 1"-RL)	✓	✓	✓	✓	
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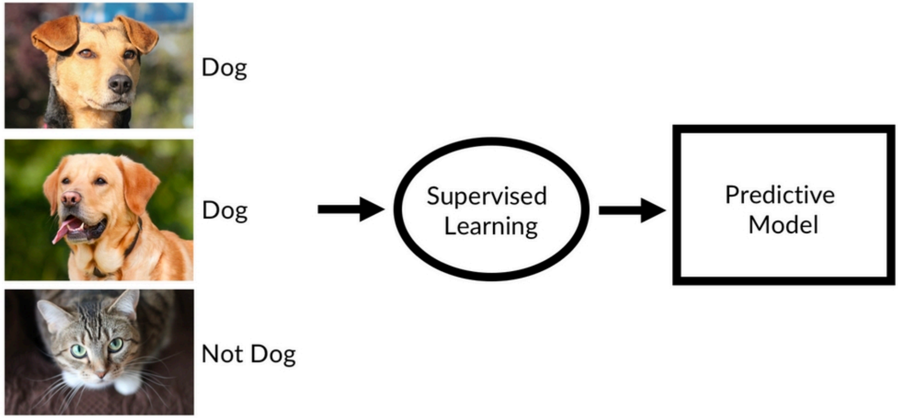

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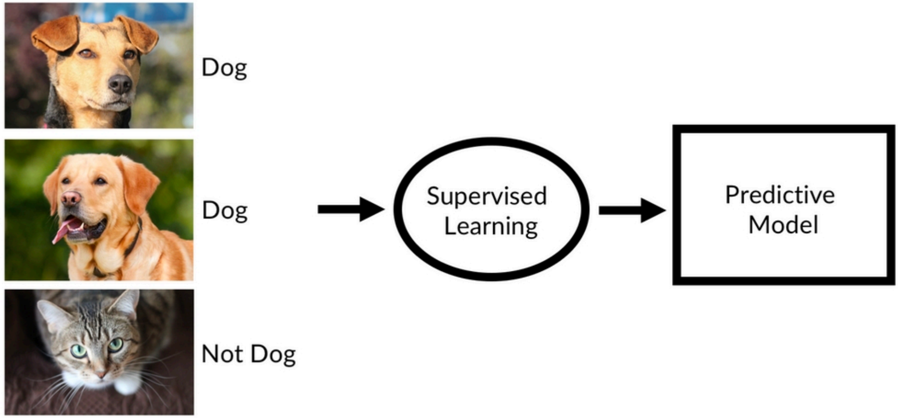

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POMDPs
Multi-Agent RL
(but MCTS)

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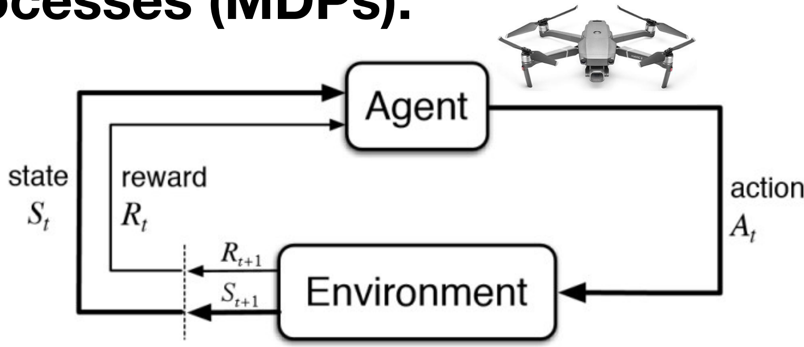

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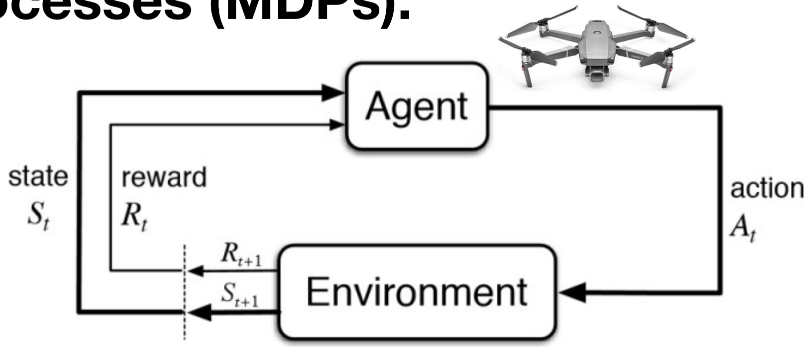

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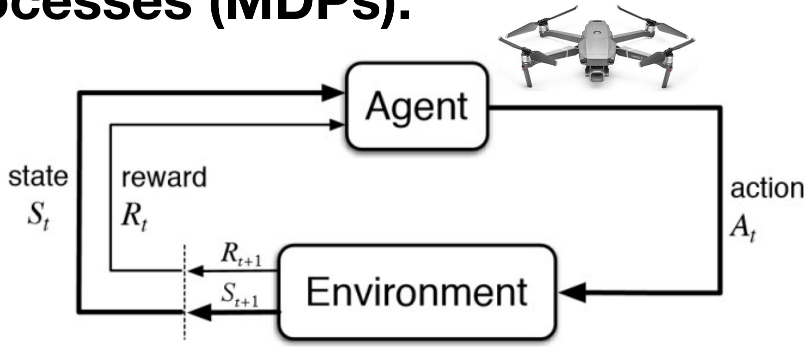

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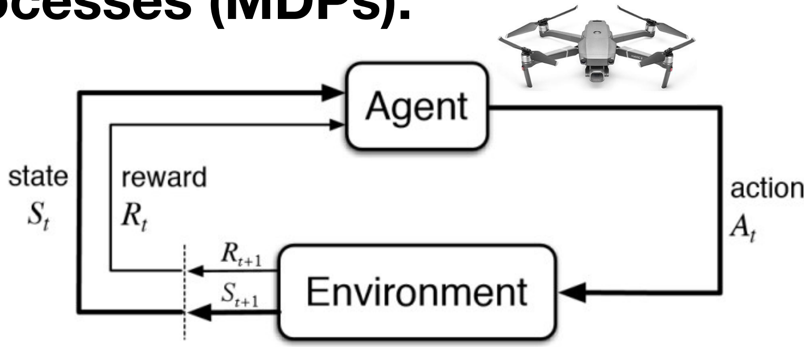

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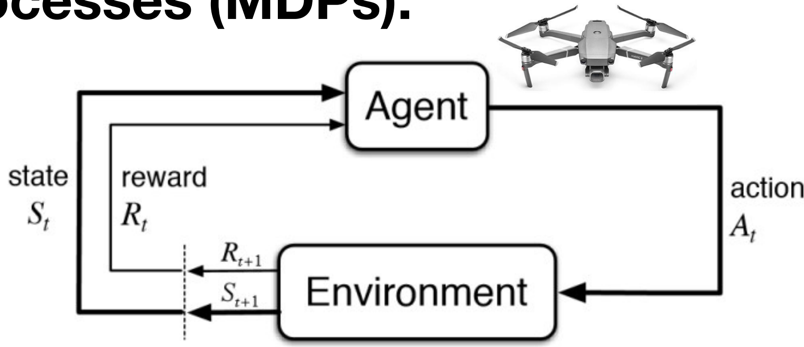

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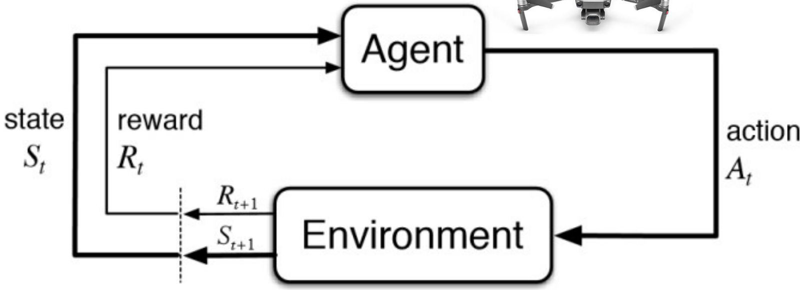

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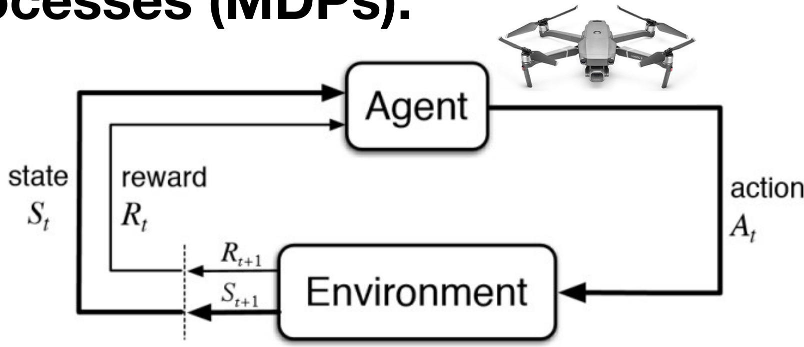

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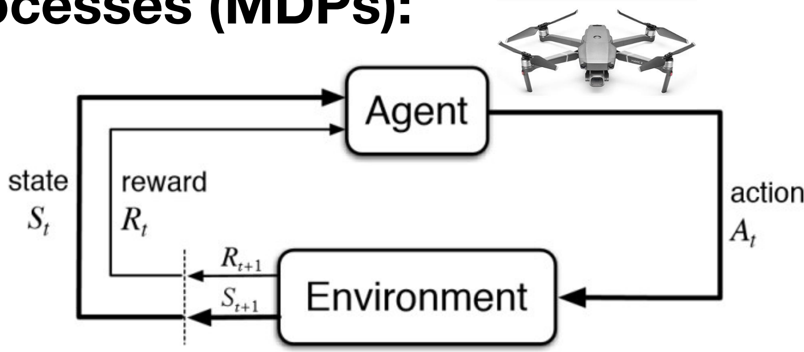

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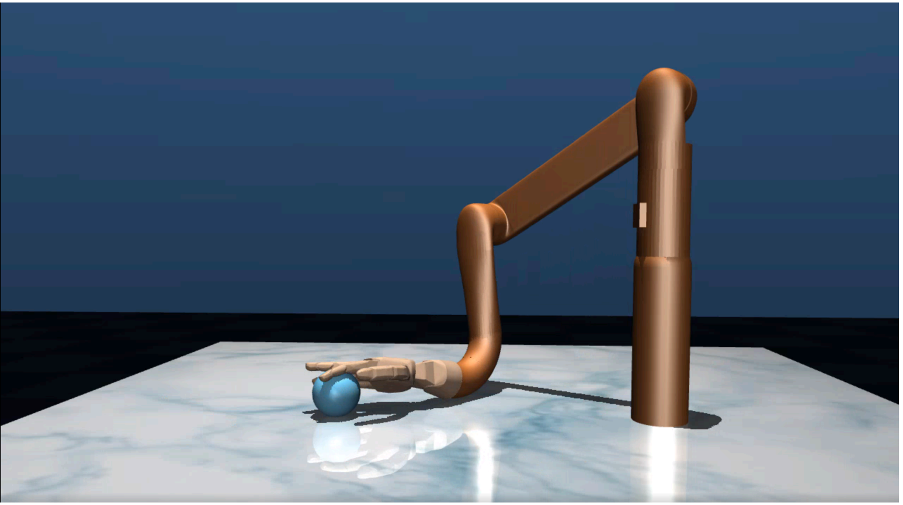
  - A time horizon $H \in \mathbb{N}$

Example:
robot hand needs to pick the ball and hold it in a goal (x,y,z) position



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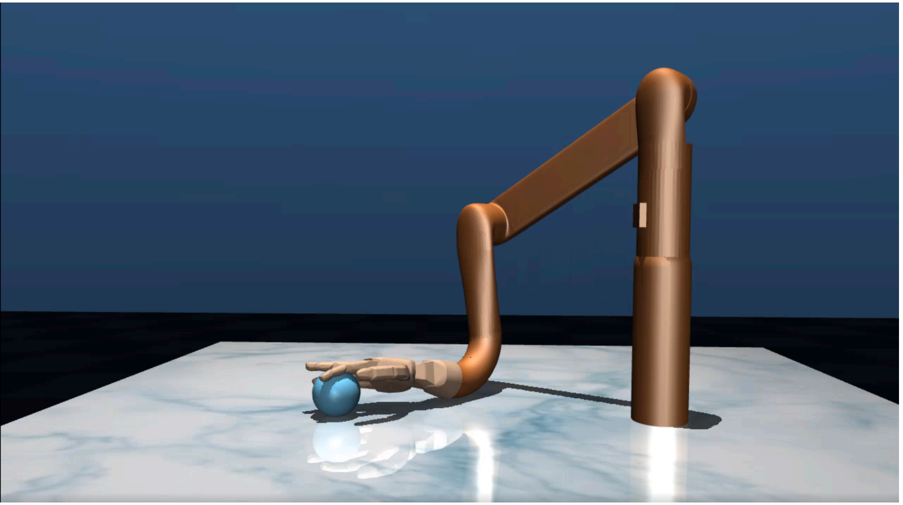

State s: robot configuration (e.g., joint angles) and the ball's position

Action a: Torque on joints in arm & fingers

Transition $s' \sim P(\cdot \mid s, a)$: physics + some noise

policy $\pi(s)$: a function mapping from robot state to action (i.e., torque)

reward/cost:

$r(s, a)$: immediate reward at state (s, a)

$c(s, a)$: torque magnitude + dist to goal

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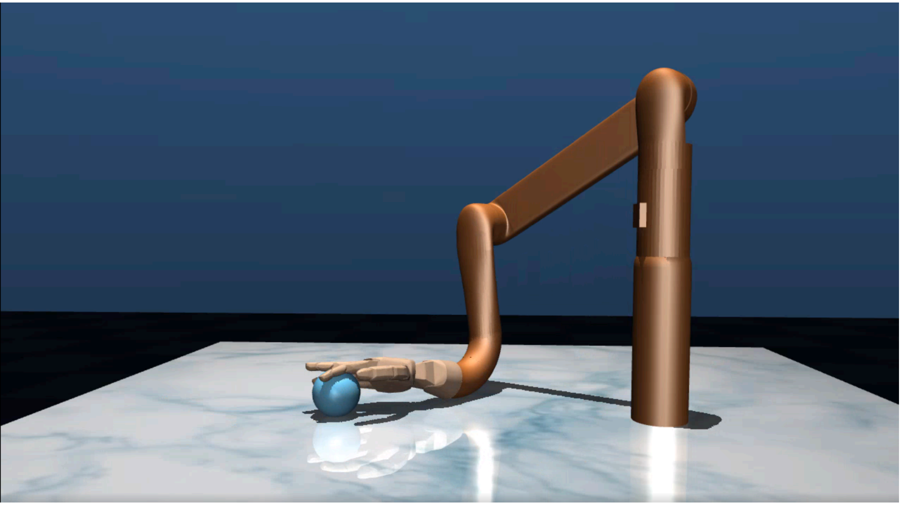

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$$\pi^\star = \arg\min_{\pi} \mathbb{E}\left[c(s_0, a_0) + c(s_1, a_1) + 2c(s_2, a_2) + \ldots c(s_{H-1}, a_{H-1}) \,\middle|\, s_0, \pi\right]$$

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or

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    - Transition to (and observe) s_{t+1} where $s_{t+1} \sim P(\cdot \mid s_t, a_t)$

$s_1 \sim P(\cdot \mid s_0, a_0)$

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  - The sampled trajectory is $\tau = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{H-1}, a_{H-1}, r_{H-1}\}$

$a_0 = \pi_0(s_0)$

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$\vdots$

$a_{H-1} = \pi_{H-1}(s_{H-1})$

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  - The rewards in this trajectory must be $r_t = r(s_t, a_t)$ (else $\rho_\pi(\tau) = 0$).

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  - Shorthand: we sometimes write ρ or ρ_π when π and/or μ are clear from context.
  - The rewards in this trajectory must be $r_t = r(s_t, a_t)$ (else $\rho_\pi(\tau) = 0$).
  - For π stochastic:
    $\rho_\pi(\tau) = \mu(s_0)\pi(a_0 \mid s_0)P(s_1 \mid s_0, a_0)\ldots\pi(a_{H-2} \mid s_{H-2})P(s_{H-1} \mid s_{H-2}, a_{H-2})\pi(a_{H-1} \mid s_{H-1})$

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- Objective: find policy π that maximizes our expected cumulative episodic reward:

$$\max_{\pi} \mathbb{E}_{\tau \sim \rho_\pi}\left[r(s_0, a_0) + r(s_1, a_1) + \ldots + r(s_{H-1}, a_{H-1})\right]$$

$|S|, |A|, H$

Today

- Logistics (Welcome!)
- Overview of RL
- Markov Decision Processes
  - Problem statement
  - Policy Evaluation

How many det. policies are there?

$S = \{1, 2, \ldots, 10\}$

a a a ... a

b a a a a

$|A|^{|S| \cdot H}$

Value function and Q functions:

Quantities that allow us to reason policy's long-term effect:

Value function and Q functions:

$V_0^{\pi}(s)$

Quantities that allow us to reason policy's long-term effect:

- Value function $V_h^\pi(s) = \mathbb{E}\left[\sum_{t=h}^{H-1} r(s_t, a_t) \,\middle|\, s_h = s\right]$

$= E[r_h + r_{h+1} + \cdots r_{H-1} \mid s_h = s]$

$a_h = \pi(s_h) \quad a_{h+1} = \pi(s_{h+1})$

Value function and Q functions:

Quantities that allow us to reason policy's long-term effect:

- Value function $V_h^\pi(s) = \mathbb{E}\left[\sum_{t=h}^{H-1} r(s_t, a_t) \,\middle|\, s_h = s\right]$

- Q function $Q_h^\pi(s, a) = \mathbb{E}\left[\sum_{t=h}^{H-1} r(s_t, a_t) \,\middle|\, (s_h, a_h) = (s, a)\right]$

$r_h + \dots \quad r_{H-1}$

$a_h = a, \quad a_{h+1} \leftarrow \pi(s_{h+1})$

Value function and Q functions: for π - det.

Quantities that allow us to reason policy's long-term effect:

- Value function $V_h^\pi(s) = \mathbb{E}\left[\sum_{t=h}^{H-1} r(s_t, a_t) \middle| s_h = s\right]$

$V_h^\pi(s)$

- Q function $Q_h^\pi(s, a) = \mathbb{E}\left[\sum_{t=h}^{H-1} r(s_t, a_t) \middle| (s_h, a_h) = (s, a)\right]$

$= Q_h^\pi(s, \pi_h(s))$

- At the last stage, what are:

$Q_{H-1}^\pi(s, a) =$ $r(s, a)$

$V_{H-1}^\pi(s) =$ $r(s, \pi_{H-1}(s))$

Value function and Q functions:

Quantities that allow us to reason policy's long-term effect:

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- At the last stage, what are:

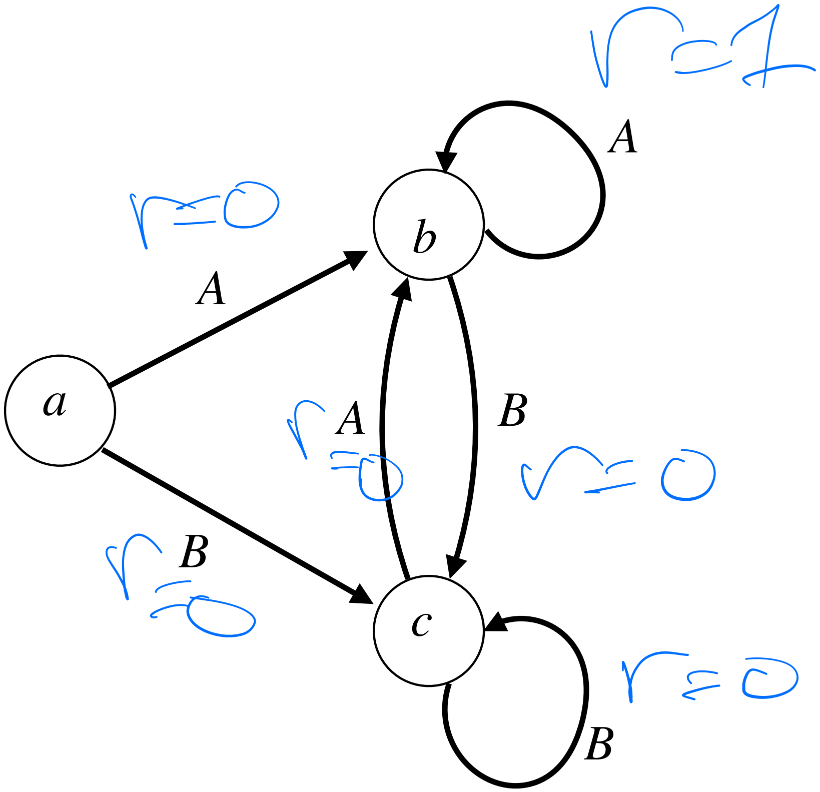
$Q_{H-1}^\pi(s, a) =$ $\qquad$ $V_{H-1}^\pi(s) =$

- Objective: (remember $s_0 \sim \mu$)

$$\max_\pi \mathbb{E}_{\tau \sim \rho_{\pi,\mu}}\left[r(s_0, a_0) + r(s_1, a_1) + \ldots + r(s_{H-1}, a_{H-1})\right] =$$

Example of Policy Evaluation (e.g. computing V^π and Q^π)

Consider the following **deterministic** MDP w/ 3 states & 2 actions, with $H = 3$



Reward: $r(b, A) = 1$, & 0 everywhere else

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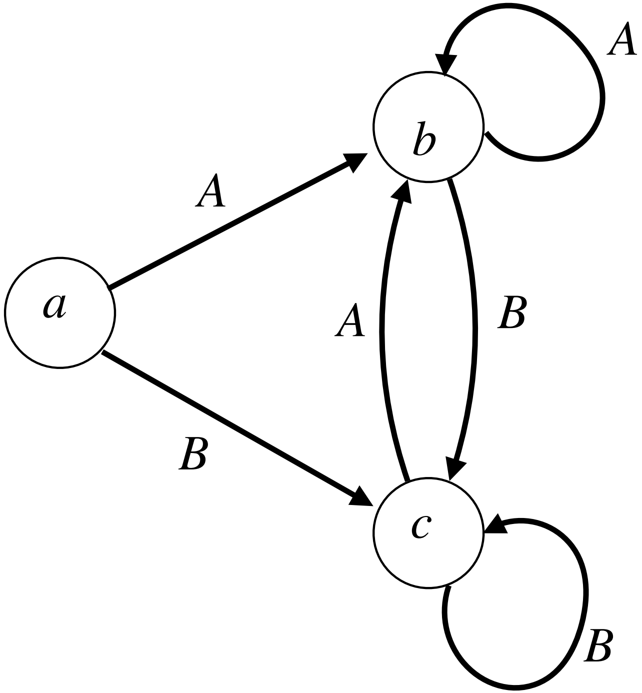

- Consider the deterministic policy $\pi_0(s) = A, \pi_1(s) = A, \pi_2(s) = B, \forall s$

Reward: $r(b, A) = 1$, & 0 everywhere else

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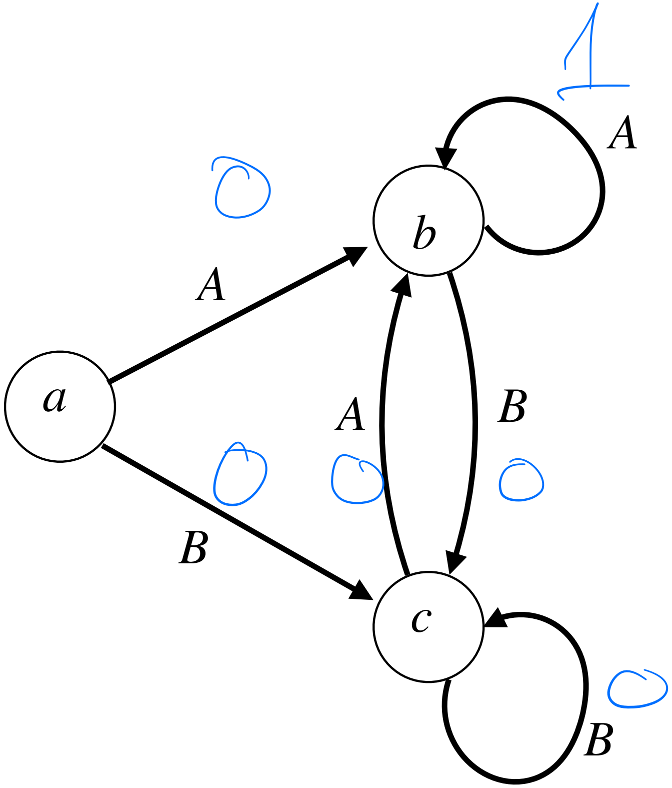

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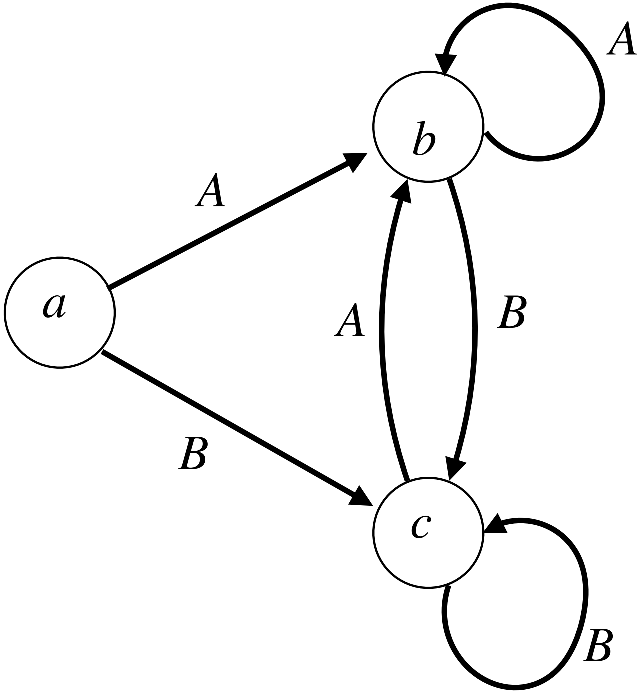

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- Consider the deterministic policy
  $\pi_0(s) = A, \pi_1(s) = A, \pi_2(s) = B, \forall s$

- What is V^π?
  $V_2^\pi(a) = 0, V_2^\pi(b) = 0, V_2^\pi(c) = 0$

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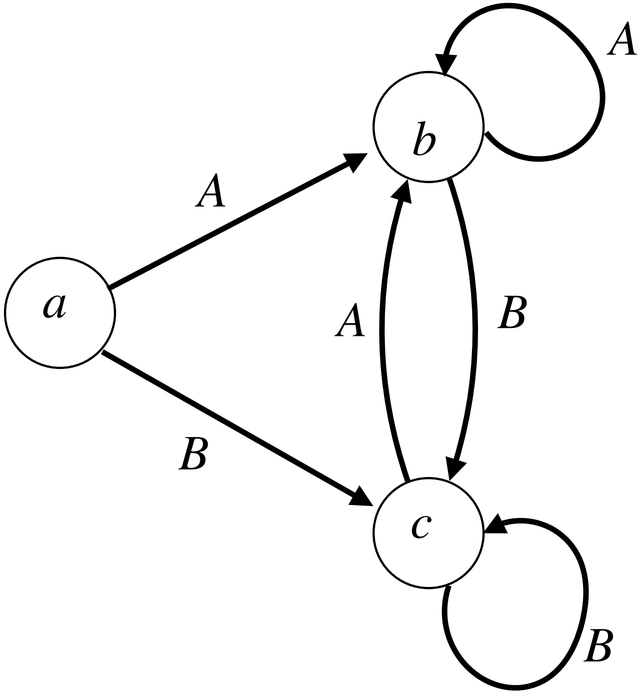

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- What is V^π?
  $V_2^\pi(a) = 0, V_2^\pi(b) = 0, V_2^\pi(c) = 0$

  $V_1^\pi(a) = 0, V_1^\pi(b) = 1, V_1^\pi(c) = 0$

  $r_1 + r_2$

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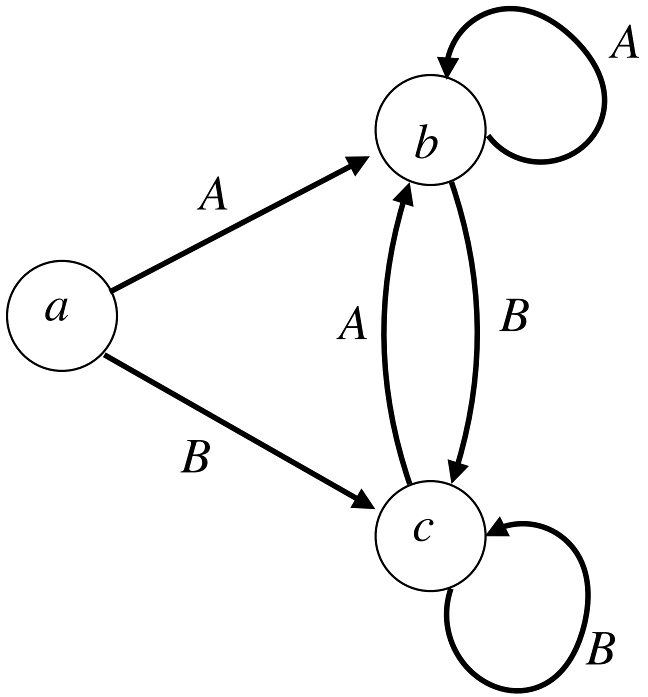

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- Consider the deterministic policy
  $\pi_0(s) = A, \pi_1(s) = A, \pi_2(s) = B, \forall s$

- What is V^π?

$$V_2^\pi(a) = 0, V_2^\pi(b) = 0, V_2^\pi(c) = 0$$

$$V_1^\pi(a) = 0, V_1^\pi(b) = 1, V_1^\pi(c) = 0$$

$$V_0^\pi(a) = 1, V_0^\pi(b) = 2, V_0^\pi(c) = 1$$

Summary:

- **Finite horizon MDPs (a framework for RL):**
- Key concepts:

  V and Q functions; sampling a trajectory $\rho_\pi(\tau)$**; ~~Bellman consistency equations~~;**