Infinite Horizon MDPs: Value and Policy Iteration

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CS/Stat 184: Introduction to Reinforcement Learning

Fall 2023

Today

- Recap
- Infinite Horizon MDPs
  - Optimality & the Bellman Equations
  - Value Iteration
  - Policy Iteration

- HW 1 is posted.
- HW 1 is long. Please start early.

Recap

Infinite Horizon MDPs:

- An MDP: $\mathcal{M} = \{\mu, S, A, P, r, \gamma\}$
  - μ, S, A, $P : S \times A \mapsto \Delta(S)$, $r : S \times A \to [0,1]$ same as before
  - instead of finite horizon H, we have a discount factor $\gamma \in [0,1)$

- Objective: find policy π that maximizes our expected, discounted future reward:

$$\max_{\pi} \mathbb{E}\left[r(s_0, a_0) + \gamma r(s_1, a_1) + \gamma^2 r(s_2, a_2) + \ldots\ldots \middle| \pi \right]$$

Value function and Q functions:

Value function and Q functions:

- Quantities that allow us to reason about the policy's long-term effect:
- Value function $V^\pi(s) = \mathbb{E}\left[\sum_{h=0}^{\infty} \gamma^h r(s_h, a_h) \,\middle|\, s_0 = s, \pi\right]$
- Q function $Q^\pi(s, a) = \mathbb{E}\left[\sum_{h=0}^{\infty} \gamma^h r(s_h, a_h) \,\middle|\, (s_0, a_0) = (s, a), \pi\right]$

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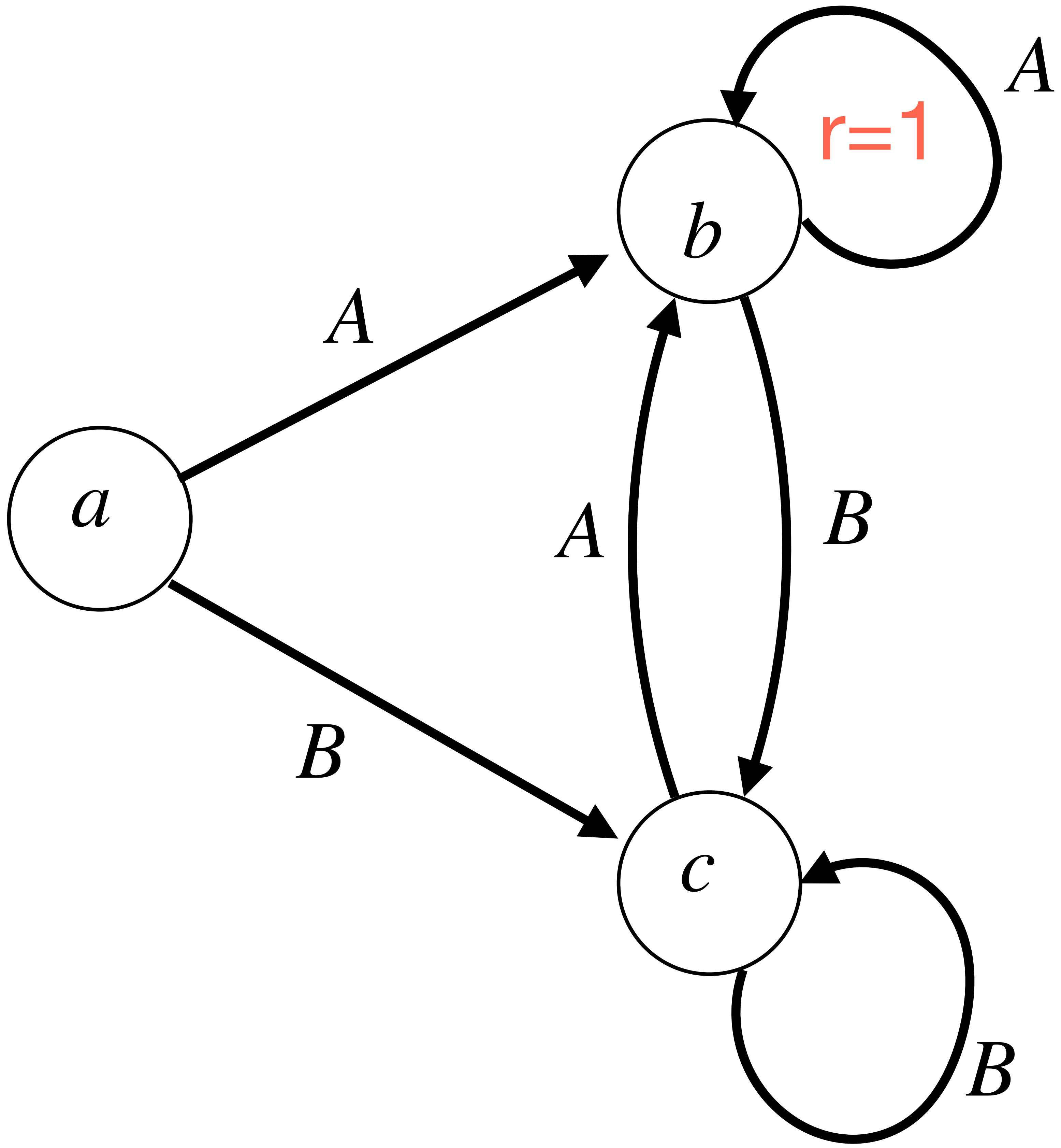
- What are upper and lower bounds on V^π and Q^π?

$$0 \le V^\pi(s), Q^\pi(s, a) \le 1/(1 - \gamma)$$

Example of Policy Evaluation (e.g. computing V^π and Q^π)

Consider the following **deterministic** MDP w/ 3 states & 2 actions



- Consider the policy
  $\pi(a) = B, \pi(b) = A, \pi(c) = A$
- What is V^π?
  $V^\pi(a) = \gamma^2/(1-\gamma)$
  $0 + 0 + \gamma^2 + \gamma^3 \ldots$

  $V^\pi(b) = 1/(1-\gamma)$

  $V^\pi(c) = \gamma/(1-\gamma)$

Reward: $r(b, A) = 1$, & 0 everywhere else

Bellman Consistency (theorem)

- Consider a fixed policy, $\pi : S \mapsto A$.
- By definition, $V^\pi(s) = Q^\pi(s, \pi(s))$
- Bellman consistency conditions:
  - $V^\pi(s) = r(s, \pi(s)) + \gamma \mathbb{E}_{s' \sim P(\cdot|s,\pi(s))}[V^\pi(s')]$
  - $Q^\pi(s, a) = r(s, a) + \gamma \mathbb{E}_{s' \sim P(\cdot|s,a)}\left[V^\pi(s')\right]$

$r(s, a, s')$

Computation of V^π

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- For a fixed policy, $\pi : S \mapsto A$, let's compute its V (and Q) value functions.
- We have the Bellman consistency conditions, for a given policy π

$$V^\pi(s) = r(s, \pi(s)) + \gamma \sum_{s'} P(s' | s, \pi(s)) V^\pi(s')$$

- How do we use this to find a solution?

Solve Linear.

- What is the time complexity?

$O(|S|^3)$

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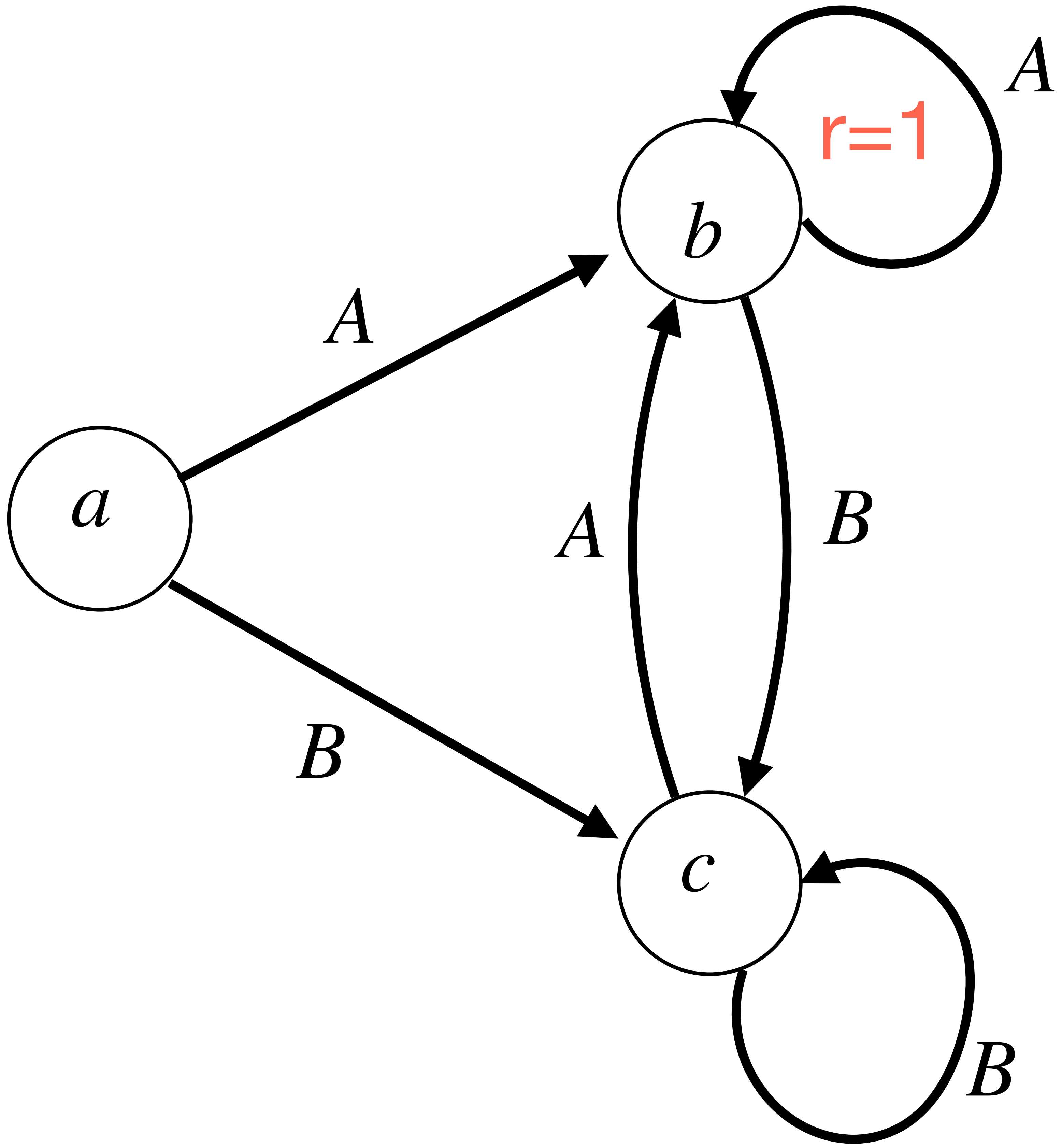
- Do you see how to write this with matrix algebra?

Let's use Bellman Consistency for computing V^π

$\pi(a) = B \qquad \pi(b), \pi(c) = A$

Consider the following **deterministic** MDP w/ 3 states & 2 actions



$V^\pi(a) = 0 + \gamma \cdot V^\pi(c)$ ✓ $\frac{\gamma^2}{1-\gamma}$

$V^\pi(b) = 1 + \gamma \cdot V^\pi(b) \Rightarrow V^\pi(b) = \frac{1}{1-\gamma}$

$V^\pi(c) = 0 + \gamma V^\pi(b)$

$\frac{\gamma}{1-\gamma}$

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Properties of an Optimal Policy $\pi^\star$

- **Theorem:** Every infinite horizon MDP has a stationary, deterministic optimal policy, that dominates all other policies, everywhere.
  - i.e. there exists a policy $\pi^\star : S \mapsto A$ such that

$$V^{\pi^\star}(s) \geq V^{\pi}(s) \quad \forall s, \ \forall \pi \in \Pi$$

(again Π is the set of all time dependent, history dependent, stochastic policies)

- $\implies$ we can write: $V^\star = V^{\pi^\star}$ and $Q^\star = Q^{\pi^\star}$.

Example of Optimal Policy $\pi^\star$, discounted case

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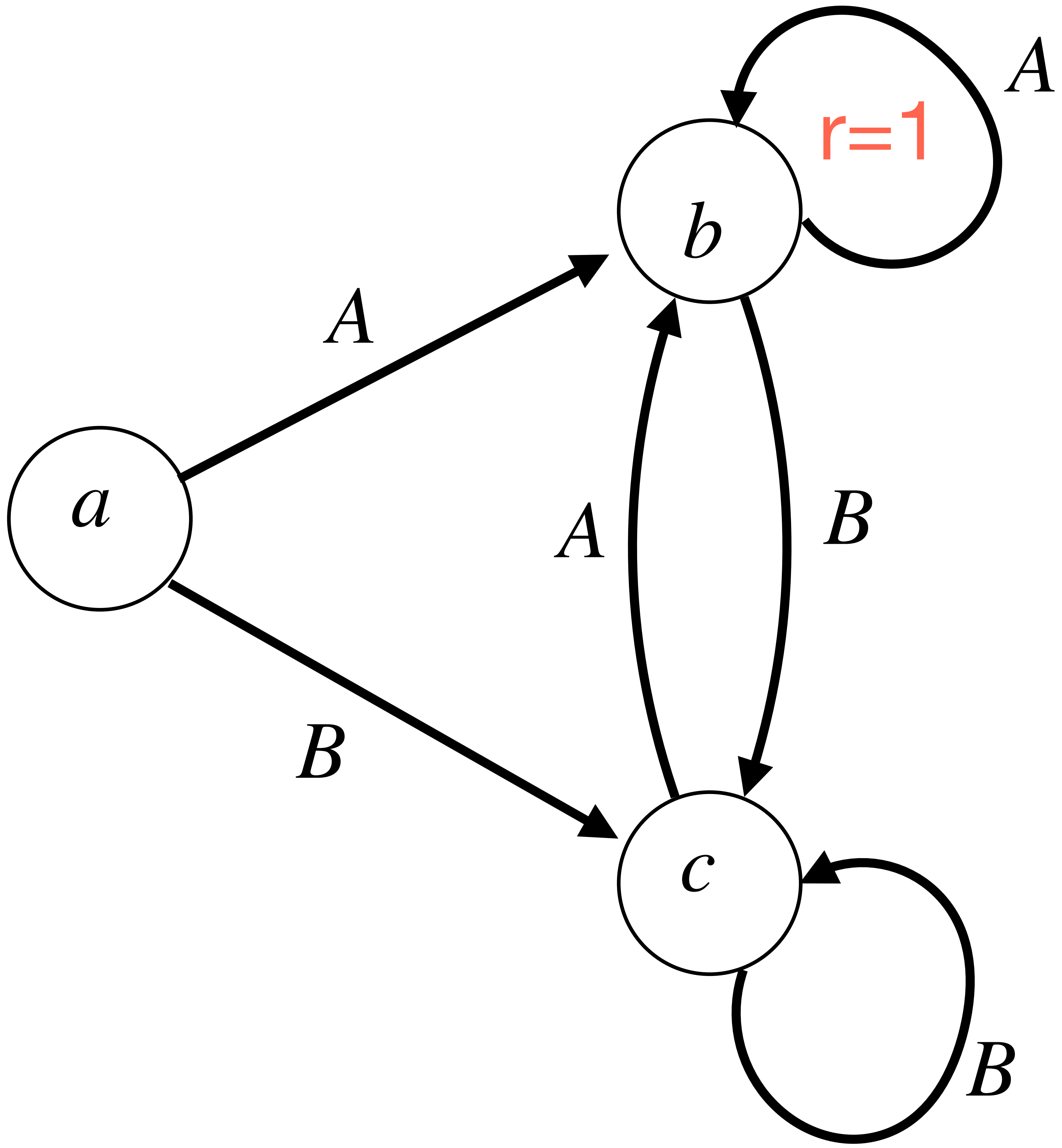

- What's the optimal policy?
  $\pi^\star(s) = A, \forall s$

- What is optimal value function, $V^{\pi^\star} = V^\star$?

$$V^\star(a) = \frac{\gamma}{1-\gamma}, \; V^\star(b) = \frac{1}{1-\gamma}, \; V^\star(c) = \frac{\gamma}{1-\gamma}$$

Reward: $r(b, A) = 1$, & 0 everywhere else

How do we compute $\pi^\star$ and $V^\star$?

- Naively, we could compute the value of all policies and take the best one.
- Suppose $|S|$ states, $|A|$ actions.
  How many different stationary polices are there? $|A|^{|S|}$

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- A function $V: S \to R$ satisfies the Bellman equations if

$$V(s) = \max_a \left\{ r(s,a) + \gamma \mathbb{E}_{s' \sim P(\cdot|s,a)} \left[V(s') \right] \right\}, \ \forall s$$

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- **Theorem:**
  - V satisfies the Bellman equations if and only if $V = V^\star$.
  - The optimal policy is: $\pi^\star(s) = \arg\max_a \left\{ r(s,a) + \gamma \mathbb{E}_{s' \sim P(\cdot|s,a)} \left[V^\star(s') \right] \right\}$.

Exercise: use the BE to the purported $\pi^\star$ is optimal

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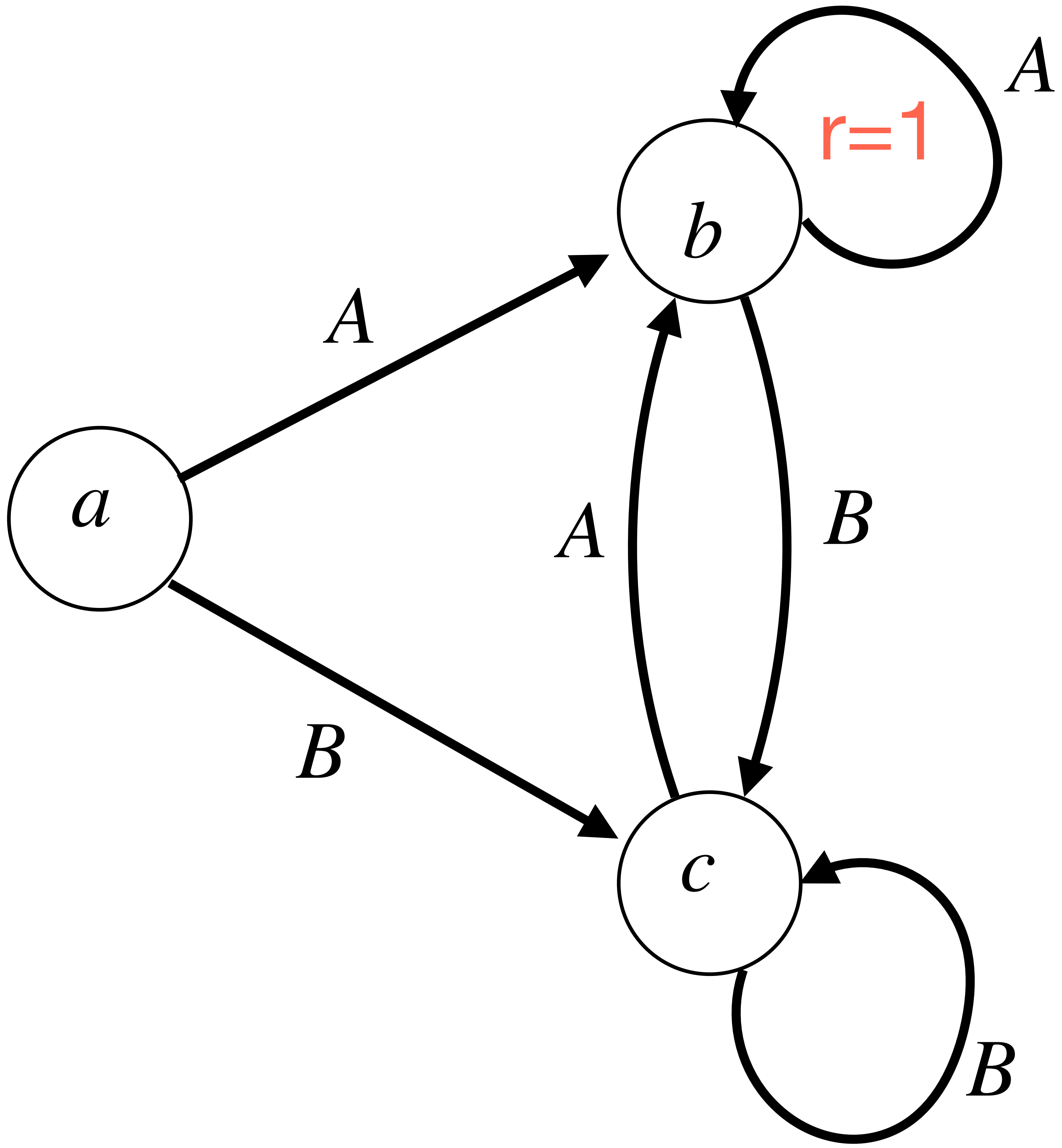

verify that

$V^{\pi^\star}(a)$ $\quad \frac{\gamma}{1-\gamma}$

$V^{\pi^\star}(b)$ $\quad \frac{1}{1-\gamma}$

$V^{\pi^\star}(c)$ $\quad \frac{\gamma}{1-\gamma}$

$\pi^\star(\cdot) = A.$

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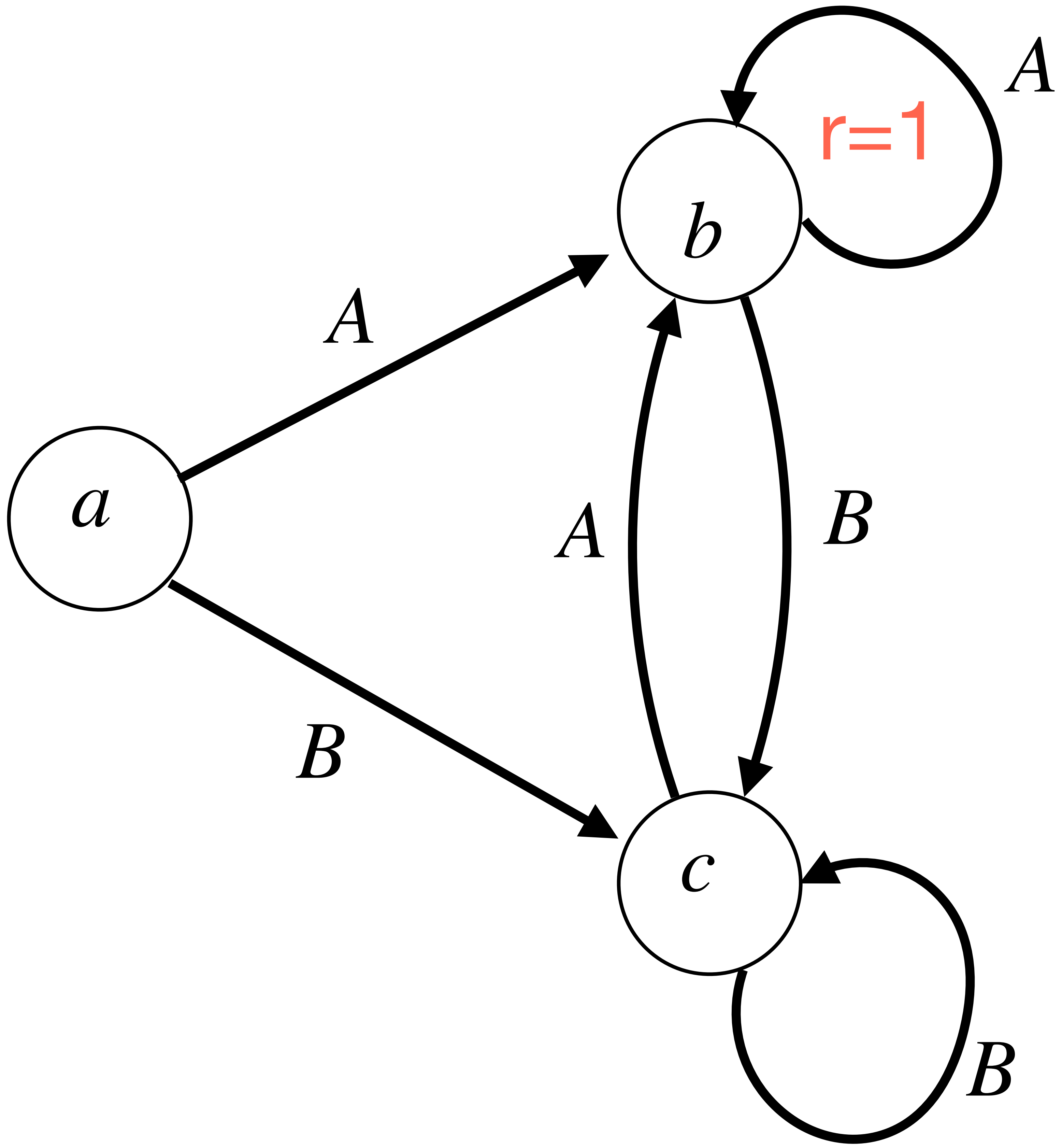

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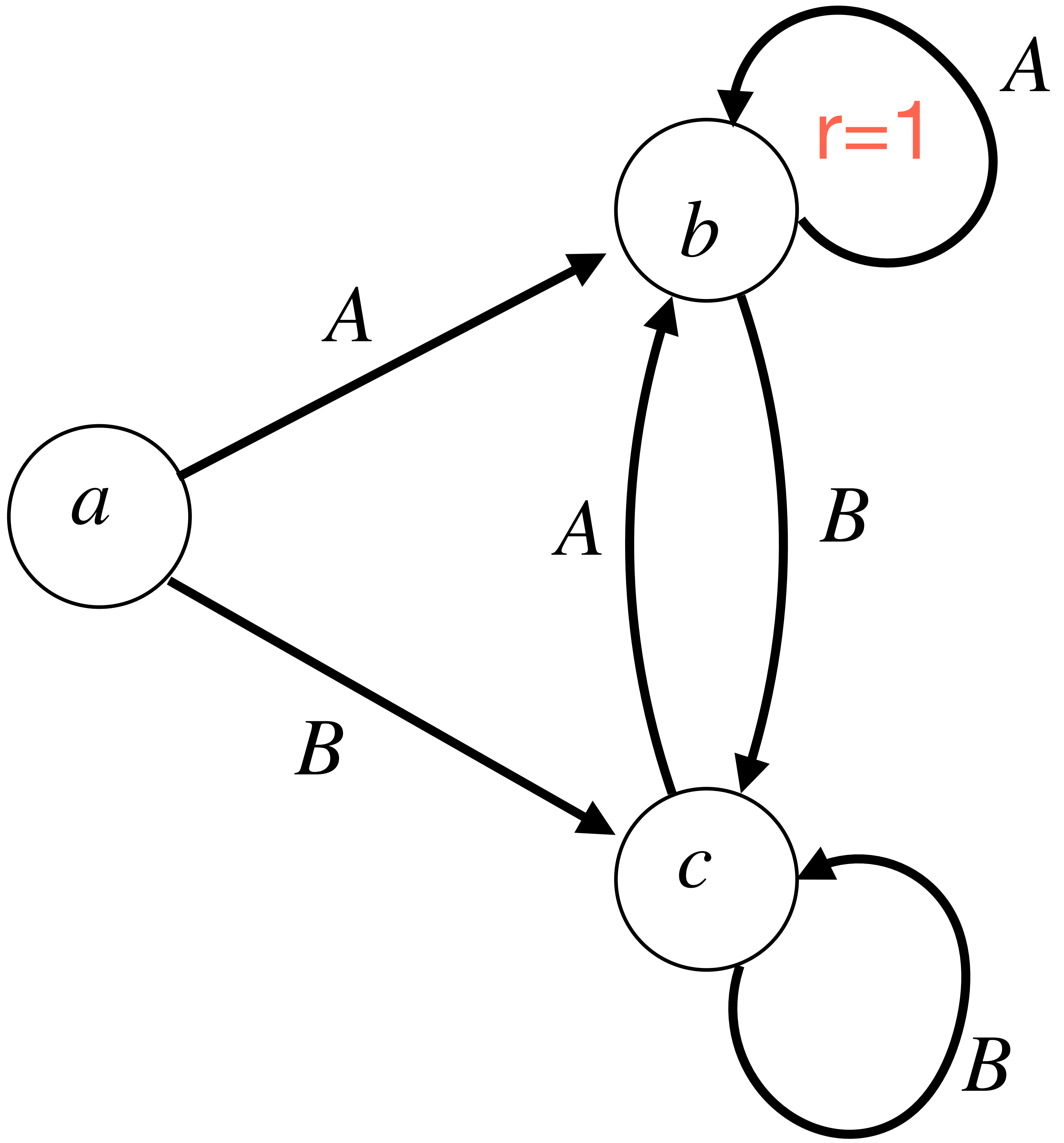

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  Then it converges, i.e. $x^t \to x^\star$, as $t \to \infty$.

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- Observe $|x^t - x^\star| = |f(x^{t-1}) - f(x^\star)| \leq \gamma |x^{t-1} - x^\star|$

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$\leq \gamma^2 |x^{t-2} - x^\star|$

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$\cdots \leq \gamma^t |x^0 - x^\star|$

- If we want $|x^t - x^\star| \leq \epsilon$, then how should we set t?

$\leq \gamma^t (b-a)$

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- If we want $|x^t - x^\star| \le \epsilon$, then how should we set t?
  - Want t such that $\gamma^t (b - a) \le \epsilon$
  - $\Longrightarrow \ t \ge \ln\big((b-a)/\epsilon\big)/(1-\gamma)$

using $t = \dfrac{\log \frac{b-a}{\epsilon}}{\log\left(\frac{1}{\gamma}\right)}$ also ok but is a more interpretable expression.

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   $$\pi(s) = \arg\max_a \left\{ r(s,a) + \gamma \mathbb{E}_{s' \sim P(\cdot \mid s,a)} V^T(s') \right\}$$

$\vec{V}^t \in \mathbb{R}^{|S|}$

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fixing s, a

$O(|S|)$

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  (assume scalar $+, -, \times, \div$ are $O(1)$ operations)

$O(|S|^2|A|)$

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- What is the per iteration computational complexity of VI?

  (assume scalar $+, -, \times, \div$ are $O(1)$ operations)
- Guarantee: VI is fix-point iteration, which contracts, so $V^t \to V^\star$, as $t \to \infty$

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  - VI then has computational complexity $O(|S|^2|A|T)$. $\approx$ $O\left(\frac{|S|^2|A|\log\frac{1}{\epsilon}}{1-\gamma}\right)$

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- Initialization: choose a policy $\pi^0 : S \mapsto A$
- For $t = 0, 1, \ldots T - 1$
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step 1 {
  - Computing V^{π^t}: solve lin system with π^t $O(|S|^3)$
  - Computing Q^{π^t} with V^{π^t}: $Q^{\pi^t}(s,a) = r(s,a) + \gamma \sum_{s'} P(s'|s,a) V^{\pi^t}(s')$ $O(|S|^2|A|)$

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  Per iteration complexity: $O(|S|^3 + |S|^2|A|)$

- What about convergence?

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  - "contraction": $\|V^{\pi^{t+1}} - V^\star\|_\infty \leq \gamma \|V^{\pi^t} - V^\star\|_\infty$

- Corollary: If we set $T = \frac{1}{1-\gamma} \ln\left(\frac{1}{\epsilon(1-\gamma)}\right)$ iterations,

  PI will return a policy π^{t+1} s.t. $\|V^{\pi^{t+1}} - V^\star\|_\infty \leq \epsilon$

  - with total computational complexity $O\Big(\left(|S|^3 + |S|^2|A|\right)T\Big)$.

Summary:

- **Discounted infinite horizon MDP:**
  - Key Concepts: Bellman equations; Value Iteration; Policy Iteration

Attendance:
bit.ly/3RcTC9T

Feedback:
bit.ly/3RHtlxy